7-1 Standardized Test Prep
Exploring Exponential Models

Multiple Choice

For Exercises 1 and 2, choose the correct letter.

1. Which of the following functions represents exponential decay and has a y-intercept of 2? **D**

    Ⓐ $y = 2\left(\frac{4}{3}\right)^x$ Ⓒ $y = \frac{1}{4}(2)^x$

    Ⓑ $y = \frac{1}{2}(0.95)^x$ Ⓓ $y = 2\left(\frac{2}{5}\right)^x$

2. Suppose you deposit $3000 in a savings account that pays interest at an annual rate of 4%. If no other money is added or withdrawn from the account, how much will be in the account after 10 years? **H**

    Ⓕ $3122.18   Ⓗ $4440.73

    Ⓖ $4994.50   Ⓘ $86,776.40

Extended Response

3. In 2009 there was an endangered population of 270 cranes in a western state. Due to wildlife efforts, the population is increasing at a rate of 5% per year.
    a. What exponential function would be a good model for this population of cranes? Explain in words or show work for how you determined the exponential function.
    b. If this trend continues, how many cranes will there be in this population in 2020? Show your work.

    [4] a. The general form of an exponential function is $y = a(b)^x$. x represents time in years, y represents the population of cranes, and a is the initial value of 270 cranes. Because the crane population is increasing by 5%, $b = 1 + r = 1 + 0.05 = 1.05$. The exponential function that models the crane population is $y = 270(1.05)^x$ OR equivalent explanation.

    b. $y = 270(1.05)^{11} \approx 461.79$; almost 462 cranes

    [3] appropriate methods and correct function, but with one computational error in evaluating the function

    [2] incorrect function or multiple computational errors in evaluating the function

    [1] correct function and population, without work shown

    [0] incorrect answers and no work shown OR no answers given

7-3 Standardized Test Prep

Logarithmic Functions as Inverses

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Which of the following is the logarithmic form of the equation $4^{-3} = \frac{1}{64}$? **C**

    Ⓐ $\log_{-3}\left(\frac{1}{64}\right) = 4$ Ⓒ $\log_4\left(\frac{1}{64}\right) = -3$

    Ⓑ $\log_{-3} 4 = \frac{1}{64}$ Ⓓ $\log_{\frac{1}{64}} 4 = -3$

2. What is the value of $\log_2 8$? **I**

    Ⓕ 64   Ⓗ 16

    Ⓖ 8   Ⓘ 3

3. How does the graph of $y = \log_5 (x - 3)$ compare with the graph of the parent function, $y = \log_5 x$? **C**

    Ⓐ translated 3 units to the left   Ⓒ translated 3 units to the right

    Ⓑ translated 3 units down   Ⓓ translated 3 units up

4. In 2009, an earthquake of magnitude 6.7 shook the Kermadec Islands off the coast of New Zealand. Also in 2009, an earthquake of magnitude 5.1 occurred in the Alaska Peninsula. How many times stronger was the Kermadec earthquake than the Alaska earthquake? **F**

    Ⓕ 39.811   Ⓗ 5.77

    Ⓖ 20.593   Ⓘ 0.025

Short Response

5. A single-celled bacterium divides every hour. The number N of bacteria after t hours is given by the formula $\log_2 N = t$.

    a. After how many hours will there be 64 bacteria?

    b. Explain in words or show work for how you determined the number of hours.

[2] a. 6 hours
    b. $\log_2 N = t$ can be written in the exponential form $2^t = N$. Substituting 64 for N, the equation becomes $2^t = 64$. Rewriting 64 with base 2, the equation becomes $2^t = 2^6$. Since the bases are equal, $t = 6$.
[1] incorrect exponential form OR incorrect explanation
[0] incorrect answers and no explanation OR no answers given

7-4

Standardized Test Prep
Properties of Logarithms

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Which statement correctly demonstrates the Power Property of Logarithms? **D**

   Ⓐ $\frac{1}{2}\log_5 9 = \log_5 81$ Ⓒ $\frac{1}{2}\log_5 9 = \log_5 18$

   Ⓑ $\frac{1}{2}\log_5 9 = \log_5 \frac{9}{2}$ Ⓓ $\frac{1}{2}\log_5 9 = \log_5 3$

2. Which expression is the correct expansion of $\log_4 (3x)^2$? **G**

   Ⓕ $\frac{1}{2}(\log_4 3 - \log_4 x)$ Ⓗ $2(\log_4 3 - \log_4 x)$

   Ⓖ $2(\log_4 3 + \log_4 x)$ Ⓘ $2\log_4 3 + \log_4 x$

3. Which expression is equivalent to $\log_7 16$? **C**

   Ⓐ $\dfrac{\log_7 16}{\log 10}$ Ⓒ $\dfrac{\log 16}{\log 7}$

   Ⓑ $\dfrac{\log_{16} 10}{\log_7 10}$ Ⓓ $\dfrac{\log 7}{\log 16}$

4. Which statement correctly expresses $4\log_3 x + 7\log_3 y$ as a single logarithm? **F**

   Ⓕ $\log_3 x^4 y^7$ Ⓗ $\log_3 (x^4 + y^7)$

   Ⓖ $\log_3 (4x + 7y)$ Ⓘ $\log_3 (4x - 7y)$

Short Response

5. The pH of a substance equals $-\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions. The concentration of hydrogen ions in pure water is 10^{-7} and the concentration of hydrogen ions in a sodium hydroxide solution is 10^{-14}.
   a. Without using a calculator, what is the difference of the pH levels of pure water and the sodium hydroxide solution?
   b. Explain in words or show work for how you determined the difference of the pH levels.

   [2] a. −7
   b. pH $= -\log[H^+]$, ΔpH $= -\log[H^+_w] - (-\log[H^+_s]) = \log[H^+_s] - \log[H^+_w]$
   $= \log[10^{-14}] - \log[10^{-7}] = -14\log 10 - (-7\log 10) = -14(1) + 7(1) = -7$
   The pH of pure water is 7 less than the pH of the sodium hydroxide solution.
   [1] incorrect equation form OR incorrect explanation
   [0] incorrect answers and no work shown OR no answers given

7-5

Standardized Test Prep

Exponential and Logarithmic Equations

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. If $9^x = 243$, what is the value of x? **C**

    Ⓐ 2   Ⓑ 5   Ⓒ 2.5   Ⓓ 10

2. If $2^{3x+2} = 64$, what is the value of x? **G**

    Ⓕ $\frac{8}{3}$ Ⓖ $\frac{4}{3}$ Ⓗ 2   Ⓘ $\frac{3}{4}$

3. If $\log(3x + 25) = 2$, what is the value of x? **A**

    Ⓐ 25   Ⓑ 75   Ⓒ $41\frac{2}{3}$ Ⓓ 100

4. Which best approximates the solution of $16^{2x} = 124$? **F**

    Ⓕ 0.869   Ⓖ 1.150   Ⓗ 1.739   Ⓘ 3.477

5. Which equation represents the solution of $2^{3x+1} = 7$? **D**

    Ⓐ $x = 3\left(\dfrac{\log 7}{\log 2} - 1\right)$ Ⓒ $x = \dfrac{1}{3}\left(\dfrac{\log 2}{\log 7} - 1\right)$

    Ⓑ $x = \dfrac{\log 7}{3\log 2} - 1$ Ⓓ $x = \dfrac{1}{3}\left(\dfrac{\log 7}{\log 2} - 1\right)$

Short Response

6. In 2007, the population of Tallahassee, Florida was 168,979. Some researchers believe that the population of Tallahassee will increase at a rate of 1% each year for the 10 years following this.

    a. If the researchers are correct, how many years will it take for the population of Tallahassee to reach 180,000?

    b. Explain in words or show your work for how you determined the number of years found in part (a).

    [2] a. about 7 years

        b. Because the population grows at a constant rate each year, an exponential model of the situation is $y = 168{,}979(1.01)^x$. $180{,}000 = 168{,}979(1.01)^x \rightarrow \frac{180{,}000}{168{,}979} = 1.01^x \rightarrow \log 1.0652 = x \log 1.01 \rightarrow x = \frac{\log 1.0652}{\log 1.01} \approx 6.348$.

    [1] incorrect number of years OR incorrect explanation

    [0] incorrect answers and no work shown OR no answers given

7-6

Standardized Test Prep

Natural Logarithms

Multiple Choice

For Exercises 1–4, choose the correct letter. Do not use a calculator.

1. What is $3 \ln 5 - \ln 2$ written as a single natural logarithm? **D**

    Ⓐ $\ln 7.5$ Ⓑ $\ln 27$ Ⓒ $\ln\left(\frac{5}{2}\right)^3$ Ⓓ $\ln 62.5$

2. What is the solution of $e^{x+1} = 13$? **G**

    Ⓕ $x = \ln 13 + 1$ Ⓖ $x = \ln 13 - 1$ Ⓗ $x = \ln 13$ Ⓘ $x = \ln 12$

3. What is the solution of $\ln(x - 2)^2 = 6$? **C**

    Ⓐ $2 + e^3$ Ⓑ $2 - e^3$ Ⓒ $2 \pm e^3$ Ⓓ $2 \pm e^6$

4. What is the solution of $e^{\frac{x}{2}+1} + 3 = 8$? **G**

    Ⓕ $x = 2 \ln 5 - 1$ Ⓖ $x = 2 \ln 5 - 2$ Ⓗ $x = 2 \ln 4$ Ⓘ $x = \frac{1}{2}(\ln 5 - 1)$

Short Response

5. The maximum velocity v of a rocket is $v = -0.0098t + c \ln R$. The rocket fires for t seconds and the velocity of the exhaust is c km/s. The ratio of the mass of the rocket filled with fuel to the mass of the rocket without fuel is R. A spacecraft can attain a stable orbit 300 km above Earth if it reaches a velocity of 7.7 km/s.

  a. What is the velocity of a spacecraft whose booster rocket has a mass ratio of 16, an exhaust velocity of 3.2 km/s, and a firing time of 40 s?

  b. Can this rocket attain a stable orbit 300 km above Earth? Explain in words or show work for how you determined your answer.

[2] a. 8.48 km/s

    b. Yes; the maximum velocity of 8.48 km/s is greater than the 7.7 km/s needed for a stable orbit. Therefore, the spacecraft can attain a stable orbit 300 km above the Earth.

[1] incorrect velocity OR incorrect explanation

[0] incorrect answers and no work shown OR no answers given