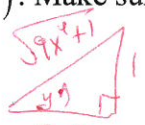
Name: KEY Date: _____

Calc H – Even MORE Practice for the test tomorrow! ☺

1. Find y' if $y = \cot^{-1}(3x^2)$. Make sure to include a triangle.

$\cot y = 3x^2$



$\csc y = \frac{\sqrt{9x^4+1}}{1} \Rightarrow \csc^2 y = 9x^4+1$

$\frac{d}{dx}[\cot y] = \frac{d}{dx}[3x^2]$

$-\csc^2 y \cdot y' = 6x$

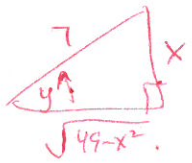
$y' = -\frac{6x}{\csc^2 y}$

$y' = -\frac{6x}{9x^4+1}$

2. Find y' if $y = \sin^{-1}\left(\frac{x}{7}\right)$ Make sure to include a triangle.

$y = \sin^{-1}\left(\frac{x}{7}\right)$

$\sin y = \frac{x}{7}$



$\frac{d}{dx}[\sin y] = \frac{d}{dx}\left[\frac{x}{7}\right]$

3. If $x = 2t^3$ and $y = t^8 + t^{-2}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

$\frac{dx}{dt} = 6t^2$

$\frac{dy}{dt} = 8t^7 - 2t^{-3}$

$\frac{dy}{dx} = \frac{8t^7 - 2t^{-3}}{6t^2} = \frac{4}{3}t^5 - \frac{1}{3}t^{-5}$

$\frac{d}{dx}\left[\frac{4}{3}t^5 - \frac{1}{3}t^{-5}\right] = \frac{\left(\frac{20}{3}t^4 + \frac{5}{3}t^{-6}\right)}{6t^2} = \frac{10}{9}t^2 + \frac{5}{18}t^{-8}$

4. a) If $x = 5 - 2t$ and $y = 8 - 5t^2$, find $\frac{dy}{dx}$

$\frac{dx}{dt} = -2$ $\frac{dy}{dt} = -10t$ $\frac{dy}{dx} = \frac{-10t}{-2} = 5t$

b) Eliminate the parameter, find an equation in x and y only.

$x = 5 - 2t$

$x - 5 = -2t$

$\frac{x-5}{-2} = t$

$y = 8 - 5\left(\frac{x-5}{-2}\right)^2$

c) Using the equation from part b, find $\frac{dy}{dx}$.

$\frac{dy}{dx} = -10\left(\frac{x-5}{-2}\right)\left(-\frac{1}{2}\right) = -\frac{5}{2}(x-5)$

d) Find the slope of the tangent line when $t = 1$. Show work to justify your answer.

$t = 1 \Rightarrow$ use $\frac{dy}{dx} = 5t$. $5(1) = 5$.

5. Let $f(x)=\begin{cases} ax^2+8x, & x\le 4 \\ bx^3, & x>4. \end{cases}$

$f'(x)=\begin{cases} 2ax+8, & x<4 \\ 3bx^2, & x>4 \end{cases}$

a = -4

Find the values of a and b such that $f(x)$ is differentiable at $x=4$.

b= $-\frac{1}{2}$

Show all work and use proper limit notation.

$\lim_{x\to 4^-}(ax^2+8x)=\lim_{x\to 4^+}(bx^3)$

$a(4)^2+8(4)=b(4)^3$

$16a+32=64b$

$a+2=4b$

$\lim_{x\to 4^-}(2ax+8)=\lim_{x\to 4^+}(3bx^2)$

$2a(4)+8=3b(4)^2$

$8a+8=48b$

$a+1=6b$

$a+1=6b$

$-(a+2=4b)$

$-1=2b$

$b=-\frac{1}{2}$

$a+2=4(-\frac{1}{2})$

$a+2=-2$

$a=-4$

6. Differentiate Implicitly with respect to x.

a) $\ln(3xy)=2y$

Rewrite to simplify:

$\ln 3+\ln x+\ln y=2y$

$\frac{d}{dx}[\ln 3+\ln x+\ln y]=\frac{d}{dx}[2y]$

$\frac{1}{x}+\frac{1}{y}\cdot y'=2y'$

$\frac{1}{x}=2y'-\frac{1}{y}y'$

$\frac{1}{x}=y'(2-\frac{1}{y})$

$\frac{1}{x}=y'(\frac{2y-1}{y})$

$y'=\frac{y}{x(2y-1)}$

sidework:

$2-\frac{1}{y}=\frac{2y}{y}-\frac{1}{y}$

$=\frac{2y-1}{y}$

b) $4x^2+4xy+3y^2=e^{2xy}$

$\frac{d}{dx}[4x^2+4xy+3y^2]=\frac{d}{dx}[e^{2xy}]$

$8x+4(y+xy')+6y\cdot y'=e^{2xy}\cdot(2(y+xy'))$

$8x+4y+4xy'+6y\cdot y'=2ye^{2xy}+2xy'e^{2xy}$

$4xy'+6yy'-2xy'e^{2xy}=2ye^{2xy}-8x-4y$

⇒ Note: $\frac{d}{dx}[xy]=y+xy'$

$y'=\frac{2ye^{2xy}-8x-4y}{4x+6y-2xe^{2xy}}$

c) $(x^2y^5)^4=\csc x$

Simplify first:

$x^8y^{20}=\csc x$

$\frac{d}{dx}[x^8y^{20}]=\frac{d}{dx}[\csc x]$

$8x^7y^{20}+20x^8y^{19}\cdot y'=-\csc x\cot x$

$20x^8y^{19}y'=-\csc x\cot x-8x^7y^{20}$

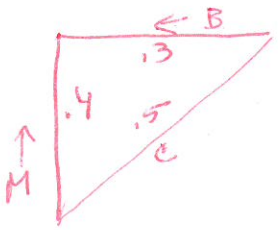
$y'=\frac{-\csc x\cot x-8x^7y^{20}}{20x^8y^{19}}$

7. Brenda's car is traveling west at 50 mi/h and Matt's car is traveling north at 60 mi/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when Brenda is 0.3 mi and Matt is 0.4 mi from the intersection?

given: $\frac{dB}{dt} = -50\ \frac{m}{h}$

$\frac{dM}{dt} = -60\ \frac{m}{h}$

$B = .3 \quad M = .4$



$(.3)^2 + (.4)^2 = (.5)^2$

Pythagorean triple... or, solve to find.

unknown: $\frac{dc}{dt}$

Formula: $M^2 + B^2 = c^2$

$\frac{d}{dt}[M^2 + B^2] = \frac{d}{dt}[c^2]$

$2M \cdot \frac{dM}{dt} + 2B \cdot \frac{dB}{dt} = 2c \cdot \frac{dc}{dt}$

$M \cdot \frac{dM}{dt} + B \cdot \frac{dB}{dt} = c\frac{dc}{dt}$

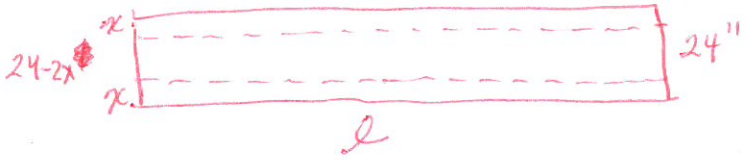
$.4(-60) + .3(-50) = .5\left(\frac{dc}{dt}\right)$

$-24 - 15 = \frac{1}{2}\left(\frac{dc}{dt}\right)$

$-78 = \frac{dc}{dt}$

They approach at 78mph.

8. A long rectangular sheet of metal, 24 inches wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?



OOPS! Not enough info to do this. Need Area or length...

Maximize $V = x(24-2x)\cdot\ell = (24x - 2x^2)\cdot\ell$

$V' = (24-4x)\cdot\ell + (24x-2x^2)\cdot\ell'$

$(24-4x)\cdot\ell + (24x-2x^2)\cdot\ell' = 0$

$(24-4x)\cdot\ell = -(24x-2x^2)\cdot\ell'$

$\frac{24-4x}{24x-2x^2} = -\frac{\ell'}{\ell}$

this is a differential equation, we haven't learned this yet! :)