Name_____ Date_____

Calculus Honors – Derivative Review 4.5-4.9 & 8.3 Period_____

Find the derivative

1. $\cos xy = 2x - 4y$

$-\sin(xy)\left(y + x\frac{dy}{dx}\right) = 2 - 4\frac{dy}{dx}$

$\frac{dy}{dx}\left(4 - x\sin(xy)\right) = 2 + y\sin(xy)$

$\frac{dy}{dx} = \frac{2 + y\sin(xy)}{4 - x\sin(xy)}$

2. $x^{3/4} - y^{3/4} = 26$

$\frac{3}{4}x^{-1/4} - \frac{3}{4}y^{-1/4}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{y^{1/4}}{x^{1/4}}$

3. $e^{2xy} = 3x - 4y$

$e^{2xy}\left(2y + 2x\frac{dy}{dx}\right) = 3 - 4\frac{dy}{dx}$

$dy/dx\left(2xe^{2xy} + 4\right) = 3 - 2ye^{2xy}$

$dy/dx = \frac{3 - 2ye^{2xy}}{2xe^{2xy} + 4}$

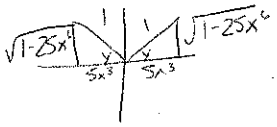
4. $3x - xy + 2x^6 - 7y^2 = 2$

$3 - y - x\frac{dy}{dx} + 12x^5 - 14y\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(x + 14y\right) = -y + 3 + 12x^5$

$dy/dx = \frac{-y + 3 + 12x^5}{x + 14y}$

5. $y = \cos^{-1}\left(5x^3\right)$

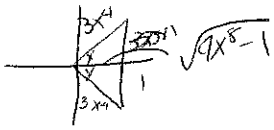


$5x^3 = \cos y$

$15x^2 = -\sin y \cdot y'$

$y' = \frac{-15x^2}{\sqrt{1-25x^6}}$

6. $y = \csc^{-1}\left(3x^4\right)$

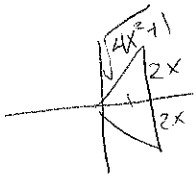


$3x^4 = \csc y$

$12x^3 = -\csc y \cot y \cdot y'$

$y' = \frac{-12x^3}{3x^4\sqrt{9x^8-1}} = -\frac{4}{x\sqrt{9x^8-1}}$

7. $y = \tan^{-1}(2x)$

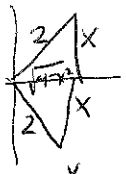


$2x = \tan y$

$2 = \sec^2 y \cdot y'$

$y' = \frac{2}{4x^2+1}$

8. $y = \left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2$



$y' = 2\left(\sin^{-1}\left(\frac{x}{2}\right)\right)\cdot\frac{1}{2\left(\frac{\sqrt{4-x^2}}{2}\right)}$

$= \frac{2\left(\sin^{-1}\left(\frac{x}{2}\right)\right)}{\sqrt{4-x^2}}$

$\sin y = \frac{x}{2}$

$\cos y \cdot y' = \frac{1}{2}$

$y' = \frac{1}{2\cos y}$

9. If $x = 15\cos t$ and $y = 15\sin t$, find $\frac{dy}{dx}$

$dx/dt = -15\sin t$

$dy/dx = -\cot t$

$dy/dt = 15\cos t$

10. If $x = 7t - 12$ and $y = 3 - 2t^2$, find $\frac{dy}{dx}$

$\frac{dx}{dt} = 7$

$dy/dx = \frac{-4t}{7}$

$dy/dt = -4t$

11. If $x = 15\cos t$ and $y = 15\sin t$, find $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = \frac{\csc^3 t}{-15\sin t} = -\frac{1}{15}\csc^3 t$

12. If $x = 7t - 12$ and $y = 3 - 2t^2$, find $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = \frac{-4/7}{7} = -\frac{4}{49}$

13. Find the values of a and b such that is differentiable at $x = 4$. Show all work and use proper limit notation.

$$f(x) = \begin{cases} ax^2 + 8x, & x \le 4 \\ bx^3, & x > 4. \end{cases}$$

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x)$

$16a + 32 = 64b$

$8a + 8 = 48b$

$16a + 32 = 64b$

$-16a - 16 = -96b$

$16 = -32b$ $\quad b = -1/2$

$f'(x) = \begin{cases} 2ax + 8 & x < 4 \\ 3bx^2 & x > 4 \end{cases}$

$\lim_{x\to 4^-} f'(x) = \lim_{x\to 4^+} f'(x)$

$8a + 8 = 48b$

$8a + 8 = -24$

$8a = -32$ $\quad a = -4$

a = -4 b = $-1/2$

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~~12.~~ Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$r = 5$

$\frac{dr}{dt} = 4$

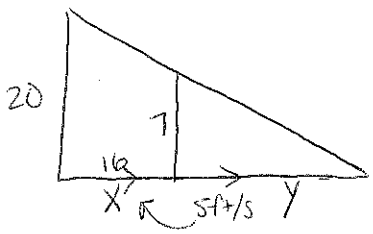
$A = 25\pi$

$\frac{dA}{dt}$

$\frac{dA}{dt} = 2\pi(5)(4) = 40\pi \text{ cm}^2/\text{min}$

15

~~13.~~ A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?



$\frac{x+y}{20} = \frac{y}{7}$

$7x + 7y = 20y$

$7x = 13y$

$7\frac{dx}{dt} = 13\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{7}{13}(5)$

$= \frac{35}{13}$ ft/s

x → distance from lamppost + person

y → length of shadow

16

~~14.~~ A container in the shape of a right circular cylinder with no top has surface area 3π ft^2. What height h and base radius r will maximize the volume of the cylinder?

$V = \pi r^2 h$

$V = \pi r^2\left(\frac{3 - r^2}{2r}\right) = \frac{3\pi}{2}r - \frac{\pi}{2}r^3$

$V' = \frac{3\pi}{2} - \frac{3\pi}{2}r^2$

$0 = \frac{3\pi}{2} - \frac{3\pi}{2}r^2$

$r = 1$

$SA = \pi r^2 + 2\pi r h$

$3\pi = \pi(r^2 + 2rh)$

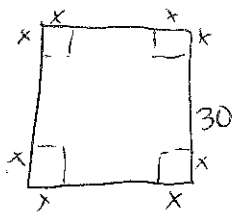
$h = \frac{3 - r^2}{2r}$

$h = \frac{3 - (3/2)^2}{2(3/2)} = \frac{3 - 9/4}{3}$

$h = 3/4 \cdot 1/3 = 1/4$

$r = 1$ ft $\quad h = 1$ ft

17 ~~15.~~ A supermarket employee wants to construct an open-top box from a 14 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?



$V = x(14 - 2x)(30 - 2x)$

$V = (14x - 2x^2)(30 - 2x)$

$V = 4x^3 - 88x^2 + 420x$

$V' = 12x^2 - 176x + 420$

$0 = 3x^2 - 44x + 105$

$x \approx 11.67$in $\quad x = 3$

3in x 3in

14

$0 < x < 7$