5.3 Factoring Polynomials HW due 1/13/14

#1–6, 39–42 Solutions

[Note: Two Methods Shown; Only Need to Do one!]

1) $x^3 - 5x^2 - 9x + 45$

Factor by Grouping:

$(x^3 - 5x^2) + (-9x + 45)$

$x^2(x-5) + -9(x-5)$

$(x-5)(x^2-9)$ ← DOTS!

$(x-5)(x+3)(x-3)$

Factor by Area Model:

	x	-5
x^2	x^3	$-5x^2$
-9	$-9x$	$+45$

$45x^3$ ✓ $45x^3$

$(x-5)(x^2-9)$

$(x-5)(x+3)(x-3)$

2) $x^3 - 2x^2 - 4x + 8$

Factor by Grouping:

$(x^3 - 2x^2) + (-4x + 8)$

$x^2(x-2) + -4(x-2)$

$(x-2)(x^2-4)$ ← DOTS!

$(x-2)(x+2)(x-2)$

Factor by Area Model:

	x	-2
x^2	x^3	$-2x^2$
-4	$-4x$	$+8$

$8x^3$ ✓ $8x^3$

$(x-2)(x^2-4)$

$(x-2)(x+2)(x-2)$

3) $x^3 + 2x^2 + 7x + 14$

Factor by Grouping:

$(x^3 + 2x^2) + (7x + 14)$

$x^2(x+2) + 7(x+2)$

$(x+2)(x^2+7)$

$\uparrow$

Sum of squares
Does not factor
with integer
terms;
stop here! ☺

Factor by Area Model:

	x	$+2$
x^2	x^3	$2x^2$
$+7$	$7x$	14

$\nearrow 14x^3$ ✓ $\searrow 14x^3$

$(x+2)(x^2+7)$

4) $3x^3 - 12x^2 + 2x - 8$

Factor by Grouping:

$(3x^3 - 12x^2) + (2x - 8)$

$3x^2(x-4) + 2(x-4)$

$(x-4)(3x^2+2)$

Factor by Area Model:

	x	-4
$3x^2$	$3x^3$	$-12x^2$
$+2$	$2x$	-8

$\nearrow -24x^3$ ✓ $\searrow -24x^3$

$(x-4)(3x^2+2)$

5) $5x^3+5x^2+x+1$

Factor by Grouping:

$(5x^3+5x^2)+(x+1)$

$5x^2(x+1)+1(x+1)$

$(x+1)(5x^2+1)$

Factor by Area Model:

	x	$+1$
$5x^2$	$5x^3$	$5x^2$
$+1$	x	1

$\nearrow 5x^3$

✓

$\searrow 5x^3$

$(x+1)(5x^2+1)$

6) $2x^3-12x^2+5x-30$

Factor by Grouping:

$(2x^3-12x^2)+(5x-30)$

$2x^2(x-6)+5(x-6)$

$(x-6)(2x^2+5)$

Factor by Area Model:

	x	-6
$2x^2$	$2x^3$	$-12x^2$
5	$5x$	-30

$\nearrow -60x^3$

✓

$\searrow -60x^3$

$(x-6)(2x^2+5)$

39) $x^3 + 512$ → Rewrite as Sum of Cubes because $8^3 = 512$

$x^3 + 8^3$ → use factoring pattern: $a = x$, $b = 8$

$(x+8)(x^2 - 8x + 8^2)$ → simplify

$(x+8)(x^2 - 8x + 64)$

40) $27x^3 - 1$ → Rewrite as Difference of cubes because $3^3 = 27$ and $1^3 = 1$

$(3x)^3 - 1^3$ → use factoring pattern: $a = 3x$, $b = 1$

$(3x-1)((3x)^2 + 1(3x) + 1^2)$ → simplify

$(3x-1)(9x^2 + 3x + 1)$

$6^3 = 3^3 \cdot 2^3$

41) $27x^3 - 216$ → Rewrite as difference of cubes because $3^3 = 27$ and $6^3 = 216$

$(3x)^3 - 6^3$ → Use Factoring Pattern: $a = 3x$, $b = 6$

$(3x-6)((3x)^2 + 6(3x) + 6^2)$ → Simplify

$(3x-6)(9x^2 + 18x + 36)$ ⟷ Notice there is a common factor of $\underline{3}$ in the linear factor and $\underline{9}$ in the quadratic factor.

$3(x-2) \cdot 9(x^2+2x+4) = 27(x-2)(x^2+2x+4)$

—OR—

Factor out the GCF:

$27(x^3 - 8)$ THEN

$27(x-2)(x^2+2x+4)$

42) $8x^3 - 27$ → Rewrite as a difference of cubes because $2^3 = 8$ and $3^3 = 27$

$(2x)^3 - 3^3$ → Use factoring pattern: $a = 2x$, $b = 3$

$(2x-3)((2x)^2 + 3(2x) + 3^2)$ → Simplify

$(2x-3)(4x^2 + 6x + 9)$