Alg 2 Final Examination Review Solutions

1) Recall inverse variation formula: $y = \frac{k}{x}$

when $d=6$, $w=120$; $d = \frac{k}{w} \Rightarrow 6 = \frac{k}{120} \Rightarrow k = 720$

what is d when $w = 108$? use the same formula with k:

$d = \frac{720}{w} \Rightarrow d = \frac{720}{108} = \boxed{6\frac{2}{3}\text{ ft.}}$

2) $\sum_{n=4}^{10}(n-1)^2 = \underbrace{(4-1)^2}_{1^{st}\text{ term}} + \underbrace{(5-1)^2}_{2^{nd}\text{ term}} + \underbrace{(6-1)^2}_{3^{rd}\text{ term}} + \dots + (10-1)^2$

$(6-1)^2 = 5^2 = \boxed{25}$

3) $V = \frac{1}{3}\pi r^2 h$ solve for r:

$\frac{V}{\frac{1}{3}\pi h} = \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi h} \Rightarrow \frac{V}{\frac{1}{3}\pi h} = r^2 \Rightarrow \frac{3V}{\pi h} = r^2 \Rightarrow \boxed{r = \sqrt{\frac{3V}{\pi h}}}$

4) Four roots (or zeros) of $f(x)$ are $3, -2, 4i, -4i$.

$\therefore$

$x = 3$, $x = -2$, $x = 4i$, $x = -4i$

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

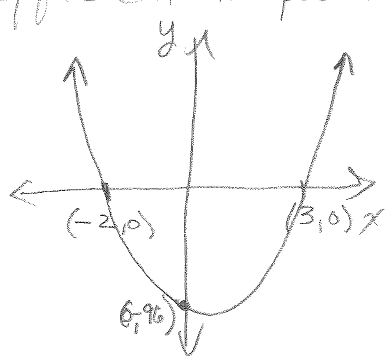
$x-3=0$ $x+2=0$ $x-4i=0$ $x+4i=0$

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

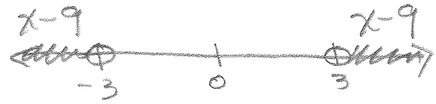
$f(x) = (x-3)(x+2)(x-4i)(x+4i)$

$f(x) = (x-3)(x+2)(x^2+16)$

Degree is 4 because there are 4 factors of x.
Quick sketch → Real zeros at 3 & −2.
y-intercept at $f(0) = (-3)(2)(16) = -96$.
Even degree so both ends same.
Lead coefficient is positive so rises to the right.



5) $|x-9| > 3$ This means, $x-9$ is more than 3 units from zero on the real number line.

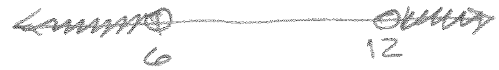


so:

$x-9 < -3$ OR $x-9 > 3$

$+9 \quad +9$ $\quad$ $+9 \quad +9$

$x < 6$ OR $x > 12$



$x < 6$ OR $x > 12$

6) $\dfrac{5x}{x-2} = 7 + \dfrac{10}{x-2}$ Domain: all Real numbers except 2.

$\cancel{(x-2)}\left(\dfrac{5x}{\cancel{x-2}}\right) = (x-2)\left(7 + \dfrac{10}{x-2}\right)$

$5x = 7x - 14 + 10$

$-2x = -4$

$x = 2$ not in domain $\therefore$ No Solution.

7) $f(x) = x^3 - 4x^2 - 3x + 12 \Rightarrow$ solve $0 < x^3 - 4x^2 - 3x + 12$

(or solve $f(x) > 0$)

Factor:

	x	-4
x^2	x^3	$-4x^2$
-3	$-3x$	12

$\Rightarrow (x^2-3)(x-4) = 0$

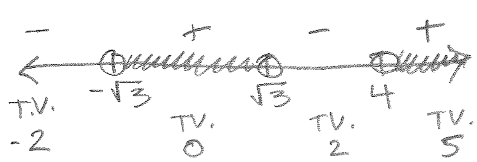
$x^2 - 3 = 0 \quad x - 4 = 0$

$x^2 = 3$

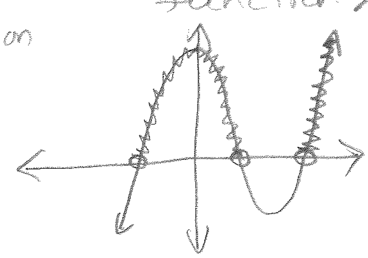
$x = \pm\sqrt{3} \quad x = 4$

Plot the zeros & choose test values between them.

Where is the function > 0?



Graph on G.C.:



$-\sqrt{3} < x < \sqrt{3}$ or $x > 4$

8) $g(x) = x^2 - 2x - 8$

x-intercepts: when $y = 0$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$0 = x - 4$ or $0 = x + 2$

$4 = x$ or $-2 = x$

x-intercepts:

$(4,0)$, $(-2,0)$

y-intercepts: when $x = 0$

$g(0) = 0^2 - 2(0) - 8 = -8$

y-intercept:

$(0,-8)$

Vertex x at $\frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$

Vertex y at $g(1) = 1^2 - 2(1) - 8 = 1 - 2 - 8 = -9$

Vertex at $(1,-9)$

Axis of symmetry is vertical line through vertex:

$x = 1$

9) $A(t) = Pe^{rt}$; $t = 5$, $P = 2500$, $r = .075$

$A(5) = 2500e^{.075(5)} = 3637.48$

Value after 5 years is \$3637.48

10) $\frac{4^{-1}x^3y^{-3}}{2^3(xy^{-2})^2} \cdot \frac{(x^3y^3)^{-1}}{(8x)^{-3}}$ ⇒ Distribute power-to-power

$\frac{4^{-1}\cancel{x^3}^{1}y^{-3}}{2^3\cancel{x^2}y^{-4}} \cdot \frac{\cancel{x^{-3}}y^{-3}}{8^{-3}\cancel{x^{-3}}}$ ⇒ Divide out common factor & move factors with negative exponents

$\frac{x\,\cancel{y^4}^{1}}{4^1 \cdot \cancel{2^3}_{8} \cdot \cancel{y^3}} \cdot \frac{8^3}{y^3}$ ⇒ Simplify numeric terms & divide out more common factors.

$\frac{x\cancel{y}}{4\cdot\cancel{8}} \cdot \frac{\cancel{8^3}8^2}{\cancel{y^3}2}$ ⇒ Divide out more common factors

$\frac{x \cdot \cancel{64}^{16}}{\cancel{4} \cdot y^2} = \frac{16x}{y^2}$

11) $\log_3(2x+3) - \log_3 4 = \log_3 x$

Domain:
$2x+3>0$
$2x>-3$
$x>-\frac{3}{2}$

$\boxed{x>0}$

apply log properties:

$$\log_3\left(\frac{2x+3}{4}\right) = \log_3 x \Rightarrow \frac{2x+3}{4} = x$$

$$4\left(\frac{2x+3}{4}\right) = 4(x)$$

$$2x+3 = 4x$$
$$3 = 2x$$
$$\frac{3}{2} = x$$ (in domain ✓)

12) $8x^3 - 125 = 0$

Use difference of cubes factoring pattern:

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$(2x)^3 - 5^3 = (2x-5)((2x)^2 + (2x)(5) + (5)^2)$$

$$(2x-5)(4x^2+10x+25) = 0$$

$2x-5=0$

$2x = 5$

$x = \frac{5}{2}$

$4x^2+10x+25=0$

$$x = \frac{-10 \pm \sqrt{100-4(4)(25)}}{2(4)}$$

$$x = \frac{-10 \pm \sqrt{-300}}{8}$$

$$x = \frac{-10 \pm 10i\sqrt{3}}{8}$$

$$x = \frac{-5 \pm 5i\sqrt{3}}{4}$$

(No #13)

14) $10^{2x-1} + 3 = 8$

Isolate the exponential expression

$$\underline{\quad -3 \quad -3}$$

$10^{2x-1} = 5$

Rewrite as a logarithm.

$\log_{10} 5 = 2x-1$

Solve for x

$$\frac{(\log_{10} 5)+1}{2} = \frac{2x}{2}$$

$x \approx 0.8$

15) a) $\log_3 3x = 2$ Rewrite as exponential. (Domain: $3x > 0 \Rightarrow x > 0$)

$3^2 = 3x \Rightarrow 9 = 3x \Rightarrow x = 3$ ✓

b) $\log_{64} x = \frac{1}{3}$ (Domain: $x > 0$)

$64^{\frac{1}{3}} = x \Rightarrow \sqrt[3]{64} = \boxed{4 = x}$ ✓

c) $x = \sqrt{2x} + 4$ Isolate the radical

$x - 4 = \sqrt{2x}$ square both sides

$(x-4)^2 = (\sqrt{2x})^2$

$x^2 - 8x + 16 = 2x$ Get zero on right & solve.

$x^2 - 10x + 16 = 0$

$(x-8)(x-2) = 0 \Rightarrow x = 8$ or $x = 2$

Check for extraneous solutions:

$8 = \sqrt{2 \cdot 8} + 4$	$2 = \sqrt{2 \cdot 2} + 4$
$8 = \sqrt{16} + 4$	$2 = \sqrt{4} + 4$
$8 = 4 + 4$	$2 = 2 + 4$
$8 = 8$ ✓	$2 \neq 6$ Extraneous

$\boxed{x = 8}$

d) $\log_3 2x + \log_3 x = \log_3 8$ Use log properties to condense

$\log_3 2x^2 = \log_3 8 \Rightarrow$ (Domain: $x > 0$)

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$ Reject -2, not in domain.

$\boxed{x = 2}$

15) (cont'd)

e) $6 \ln x = 10$ Isolate log expression

$\ln x = \frac{10}{6} = \frac{5}{3}$ Rewrite as exponential (Recall: $\ln \equiv \log_e$)

eval. in calculator $\rightarrow$ $e^{5/3} = x \approx 5.29$

f) $\sqrt[4]{1-2x} = 2$ Raise both sides to 4^{th} power

$\left(\sqrt[4]{1-2x}\right)^4 = 2^4$

$1-2x = 16$

$-2x = 15$

$x = -\frac{15}{2}$

Check:

$\sqrt[4]{1-2\left(-\frac{15}{2}\right)} = 2$

$\sqrt[4]{1+15} = 2$

$\sqrt[4]{16} = 2$

$2 = 2$ ✓

g) $x^2 - 9x < 22$ Polynomial inequality: get zero on right.

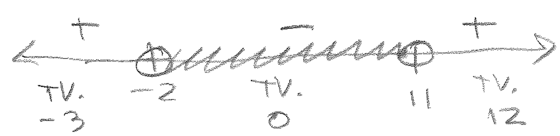
$x^2 - 9x - 22 < 0$

$x^2 - 9x - 22 = 0$

$(x-11)(x+2) = 0$

$x = 11$ or $x = -2$

Find zeros on real number line & choose test values: Where is it less than zero?



$-2 < x < 11$

16) $\frac{1}{3}$ is a zero: Use synthetic division to get the depressed (remaining) polynomial:

$$\begin{array}{c|cccc} \frac{1}{3} & 3 & 5 & -47 & 15 \\ & & 1 & 2 & -15 \\ \hline & 3 & 6 & -45 & \boxed{0} \end{array}$$

$3x^2 + 6x - 45$ $\rightarrow$ factor out common factor

$3(x^2 + 2x - 15)$

$3(x+5)(x-3)$