Name: ___________________________________ Date:_____________

Algebra 2 – Summary of Exponential Models

❶ Interest compounded n times per year: r is annual rate in decimal, P is the initial investment principal, A is the amount after t years, and t is the number of years the money is invested.

$A = P\left(1+\frac{r}{n}\right)^{nt}$ ← Note: r is part of the ***BASE*** (growth or decay factor)

r is negative for decreasing (decay) models

❷ Interest compounded *continuously*:

$A = Pe^{rt}$ ← Note: r is in the ***EXPONENT***;

r is negative for decreasing (decay) models

❸ Basic Exponential Models:

$f(x) = Ca^x$

C is the INITIAL AMOUNT
When a is **greater than 1**, it is a **GROWTH** model
When a is **between 0 and 1**, it is a **DECAY** model
Horizontal asymptote is $y = 0$
Domain is all Real numbers
Range is $y > 0$

❹ Natural Base Models:

$f(x) = Ce^{bx}$

C is the INITIAL AMOUNT
When b is **greater than 0**, it is a **GROWTH** model
When b is **less than 0**, it is a **DECAY** model
Horizontal asymptote is $y = 0$
Domain is all Real numbers
Range is $y > 0$

❺ Half-Life Decay:

$A(t) = A_0\left(2^{-\frac{t}{h}}\right)$

A_0 is the INITIAL AMOUNT, h is the half-life in periods

and t is the number of periods is has been decaying, and $A(t)$ is the amount remaining after t periods.

Exponential Model Examples:

1) The amount of money, A, after t years for \$4000 invested at 6% annual interest, compounded quarterly, is given by:

$$A = 4000\left(1 + \frac{0.06}{4}\right)^{4t}$$

2) \$2500 is invested at 5.5% annual interest, compounded continuously for t years:

$$A = 2500e^{0.055t}$$

3a) A \$1500 diamond ring purchased in 2004 increases in value by 7% per year from year t, where t = 0 represents the year 2004:

$A = 1500(1.07)^t$ ⟵ base is > 1; GROWTH model

3b) A \$20,000 car purchased in 1998 decreases in value by 14% each year from year t, where t = 0 represents the year 1998:

$A = 20000(.86)^t$ ⟵ base is < 1; DECAY model

Note: For the next two examples, the information given is *not sufficient* to set up the equation. The examples are given with the formula to illustrate the growth or decay format of an equation. For problems of this nature, the formula will be provided, and the problem will require you to solve for a specific year (*t*) or some other feature.

4a) The population growth model for Colonial America from 1610 to 1780 is given by the formula:

$P = 242.4e^{0.00008t^2}$ ⟵ exponent coefficient is > 0; GROWTH model

4b) The radioactive decay of 100 grams of radium in a container is given by the formula:

$R = 100 \cdot \left(2^{-0.00062t}\right)$ ⟵ exponent coefficient is < 0 and base is >1;
DECAY model