Algebra 2 – Sequences & Series Unit Name: ___________________________ Date: ______
Notes – Geometric Series

Geometric Series

Now that you know a series is the sum of terms in a sequence, it should be comfortable to recognize that a geometric series is the sum of terms in a geometric sequence.

Formula

When the sequence is geometric, we have two formulas: one for a **finite sum** and one for an **infinite sum**. How can there be an infinite sum, you might ask? (Please ask!)

If the common ratio of a geometric series is **greater than -1 and less than 1,** then there is a sum for the **infinite series**. Think about this: we keep adding terms that are shrinking to zero. Eventually, it is like we are just adding zero, so we say the series **converges**. (That is an awesome math word, by the way!)

Formula for a finite geometric series:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

where n is the number of terms to add up, a_1 is the first term in the sum and r is the common ratio.
Note that this formula ONLY works for a finite geometric series!

Formula for an infinite geometric series:

$$S = \frac{a_1}{1-r}$$

Only when $-1 < r < 1$

where a_1 is the first term in the sum and r is the common ratio.
Note that this formula ONLY works for an infinite geometric series that converges!

Example of an infinite geometric series:

$$\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$$

Skills

❶ Evaluate a finite geometric series given sigma notation.

Technique: Determine the number of terms, the first term and the common ratio. Then use the formula for a finite geometric series.

Example:

Evaluate the geometric series described:

$$\sum_{n=1}^{10} 4\left(\frac{2}{5}\right)^{n-1}$$

Solution:

We want to use the formula

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

First find the number of terms. To find the number of terms in the sum, subtract the starting number from the ending number and then add 1. $10 - 1 + 1 = 10$. There are 10 terms in this sum, so in our formula, $n = 10$.

Next, we need the first term, a_1. The first term in this series is $4\left(\frac{2}{5}\right)^{1-1} = 4\left(\frac{2}{5}\right)^{0} = 4(1) = 4$

So, $a_1 = 4$.

Next, we need the common ratio. This is the base, so $r = \frac{2}{5}$.

Now, we can substitute:

$$S_{10} = \frac{4\left(1 - \left(\frac{2}{5}\right)^{10}\right)}{1 - \left(\frac{2}{5}\right)} = 6.66597$$

(Use a calculator for the above.)

❷ Evaluate an infinite geometric series given sigma notation.

Technique: Verify that the series converges. Then determine the first term and the common ratio. Then use the formula for an infinite geometric series.

Example:

Evaluate the infinite geometric series described:

$$\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$$

Solution:

We want to use the formula

$$S = \frac{a_1}{1-r}$$

First, verify that there is a sum. The common ratio is $\frac{1}{2}$, which is between -1 and 1, so ther series converges.

Next, we need the first term, a_1. The first term in this series is $3\left(\frac{1}{2}\right)^{1-1} = 3\left(\frac{1}{2}\right)^{0} = 3(1) = 3$
So, $a_1 = 3$.

Now, we can substitute:

$$S = \frac{3}{1-\left(\frac{1}{2}\right)} = \frac{3}{\frac{1}{2}} = 6$$

✐ Now, practice the odd-numbered problems on the 9-5 practice sheet.