

Chapter 9.3 – Geometric Sequences– Notes

In our last class we learned about arithmetic sequences. We learned to identify these sequences by the particular pattern of having a common difference between consecutive terms.

In this section, we will learn about a particular type of sequence called a **geometric sequence**.

Exploration

In what way are these sequences like each other?

- a) 1, 2, 4, 8, 16, ...
- b) 2, -6, 18, -54, 162, ...
- c) 10, 100, 1000, 10000, 100000, ...
- d) 10, 1, 0.1, 0.01, 0.001, ...

Describe the characteristics that all of the above sequences share:

If you described this correctly, then you know how to identify a **geometric sequence**!

A **geometric sequence** is a sequence in which the ratio of each pair of consecutive terms is constant. We call this the **common ratio**, denoted as r . We find r by dividing:

$$r = \frac{a_2}{a_1}$$

Actually we can divide any term by the term that precedes it to find r :

$$r = \frac{a_n}{a_{n-1}}$$

Notice that we **multiply by the common ratio** to get the next term. When the terms alternate signs, the common ratio is negative.

Like other sequences, geometric sequences can be expressed by a recursive formula or by an explicit formula.

Recursive Formula for a Geometric Sequence:

$$\begin{aligned}a_1 &= a \\ a_n &= a_{n-1}(r)\end{aligned}$$

Where a is the first term, r is the common ratio, and $n > 1$.

Although we can use a recursive formula, the explicit formula is more versatile and the one we will usually use for these sequences.

Explicit Formula for a Geometric Sequence:

$$a_n = a_1(r)^{n-1}$$

Where a_1 is the first term, r is the common ratio, and $n \geq 1$.

Another form of the explicit formula is very useful if you happen to know two terms of the sequence, but not the first term.

Alternate Explicit Formula for a Geometric Sequence:

$$a_n = a_k(r)^{n-k}$$

Where a_k is a known k^{th} term, r is the common ratio, and $n \geq 1$.

✎ Follow the examples we do in class to see how these formulas are used.

✎ Now try the odd-numbered problems in the 9-3 Practice worksheet.