

**Chapter 9.2 – Arithmetic Sequences– Notes**

In our last class we were introduced to sequences. We learned two types of formulas for sequences: **recursive** and **explicit**.

In this section, we will learn about a particular type of sequence called an **arithmetic sequence**.

(Note: when we use the word **arithmetic** to describe a sequence, it is pronounced differently than when we say it to mean the math of operating on numbers; the third syllable is emphasized: “**a-rith-MET-ic**”.)

Exploration

In what way are these sequences like each other?

a) 1, 5, 9, 13, 17, ...

b) 0, -3, -6, -9, -12, ...

c) 100, 200, 300, 400, 500, ...

d)  $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \dots$

Describe the characteristics that all of the above sequences share:

Now determine which of the following sequences share this same characteristic:

e) 4, -4, -12, -20, -28, ...

f) 1, 2, 4, 8, 16, ...

g) 0, 1, 3, 6, 10, ...

h) 10, 9, 8, 7, 6, ...

i) 5, 25, 45, 65, 85, ...

If you chose correctly, then you know how to identify an **arithmetic sequence**!

An **arithmetic sequence** is a sequence in which the difference between each pair of consecutive terms is constant. We call this the **common difference**, denoted as  $d$ . We find  $d$  by subtracting:

$$d = a_2 - a_1$$

Actually we can subtract any term from the term that follows to find  $d$ :

$$d = a_n - a_{n-1}$$

Notice that we **add the common difference** to each term to get the next term. When the terms decrease, the common difference is negative.

Like other sequences, arithmetic sequences can be expressed by a recursive formula or by an explicit formula.

**Recursive Formula** for an Arithmetic Sequence:

$$\begin{aligned}a_1 &= a \\ a_n &= a_{n-1} + d\end{aligned}$$

Where  $a$  is the first term,  $d$  is the common difference, and  $n > 1$ .

Although we can use a recursive formula, the explicit formula is more versatile and the one we will usually use for these sequences.

**Explicit Formula** for an Arithmetic Sequence:

$$a_n = a_1 + (n - 1)(d)$$

Where  $a_1$  is the first term,  $d$  is the common difference, and  $n \geq 1$ .

Another form of the explicit formula is very useful if you happen to know two terms of the sequence, but not the first term.

**Alternate Explicit Formula** for an Arithmetic Sequence:

$$a_n = a_k + (n - k)(d)$$

Where  $a_k$  is a known  $k^{\text{th}}$  term,  $d$  is the common difference, and  $n \geq 1$ .

👉 Follow the examples we do in class to see how these formulas are used.

✍️ Now try the odd-numbered problems in the 9-2 Practice worksheet.