Algebra 2 Name: ___________________________ Date: _________

Chapter 9 - Sequences and Series – Notes

In this unit, (our last unit of the year!), we will study a type of function called a **sequence** and we will work with sequences in various ways.

What we will learn this unit:

9.1 Sequences and their formulas
9.2 Arithmetic Sequences
9.3 Geometric Sequences
9.4 Arithmetic Series
9.5 Geometric Series

While some of what we learn will build on previous concepts, there is also some new notation that we will learn in this unit. Understanding the notation is **critical** to success in this unit! Please **ask questions** if the notation is not clear to you.

Before we begin the first section, a reminder of a few concepts we have defined earlier in the course:

- Recall that the **natural numbers** are the counting numbers: $1, 2, 3, 4, \ldots$
- Recall that a **relation** is a rule that creates ordered pairs of an input and an output.
- Recall that a **function** is a relation where the input values do not repeat.

With these ideas in mind, let's look at sequences.

<u>Sequences</u>

A **sequence** is a function that relates each of the natural numbers to an output term.

Sequences can be expressed in several ways. One way is just to list terms, separated by commas, until the pattern is clear, and then indicate that the pattern continues.

Examples:

a) $1, 2, 3, 4, 5, \ldots$
This is the sequence of the natural numbers themselves! 1 is the first term, 2 is the second term, etc.

b) $2, -2, 2, -2, 2, -2, \ldots$
This is the sequence of alternating 2 and -2. The first term is 2, the second term is -2, and so on. Notice that all the odd-numbered terms are 2 and all the even-numbered terms are -2.

c) $-5, 0, 5, 10, 15, 20, \ldots$
This is the sequence that starts with -5 and then each term is 5 more than the term before.

d) $1, 1, 2, 3, 5, 8, 13, 21, \dots$
This is the sequence that starts with 1,1 and then each subsequent term is the sum of the last two terms.
This is actually a very famous sequence called the Fibonacci sequence.

e) $2, 4, 8, 16, 32, 64, \dots$
This is the sequence that starts with 2 and each term is two times the term before. Notice that this sequence lists the powers of 2, starting with the first power of 2.

f) $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$
How would you describe this sequence? What is the 100th term of this sequence?

In the last example, you were able to use the pattern to "jump" to a particular term of a sequence without necessarily figuring out all the terms that came before it. This is something we often want to know about a sequence. When we want to know a particular term, we identify it as "the 4th term" or "the 19th" term or whatever term we want. To generalize this idea, we use the expression " $\boldsymbol{n^{th}}$ term".

This brings us to another way to express a sequence. If there is an algebraic formula that can give us any term we want just by substituting the particular term we want, we call this an **explicit** $\boldsymbol{n^{th}}$ term formula.

Notation:

In order to know we are expressing a sequence and not a real-valued function, we use a different notation.

The **input** of the function is denoted $\boldsymbol{n}$, and you can think of it (informally) as the number-position of the term. That is, for the *first* term, $n = 1$. For the *second* term, $n = 2$, and so on. So, for the fiftieth term, $n = 50$. This allows us to "jump" to a particular term that we want.

The **output** of the function is the **term** itself, and it is denoted $\boldsymbol{a_n}$. The formula itself is going to be an algebraic expression with n.

Consider the sequence: $a_n = n + 2$

We get the list of terms by substituting a 1 for n to get the first term:

$a_1 = 1 + 2 = 3$ So, the first term is 3. That is, $a_1 = 3$.
$a_2 = 2 + 2 = 4$ So, the second term is 4. That is, $a_2 = 4$.
$a_3 = 3 + 2 = 5$ So, the third term is 5. That is, $a_3 = 5$.

The sequence generated by this formula is $3, 4, 5, 6, 7, 8, \ldots$

Note that the n in the subscript of a_n is the position of the term in the sequence, and the value of a_n is the output term itself.

Examples:

a) $a_n = n$
This formula pairs each natural number with itself! The first term is 1, the second term is 2, etc. This is the same sequence as in the previous example a) above.

b) $a_n = 2(-1)^{n+1}$
This is the sequence of alternating 2 and -2.

c) $a_n = -5 + (n - 1)(5)$
This is the sequence that starts with -5 and then each term is 5 more than the term before.

d) $a_1 = 1$
$a_2 = 1$
$a_n = a_{n-1} + a_{n-2}$
This is the sequence that starts with 1,1 and then each subsequent term is the sum of the last two terms.

Notice how this last sequence cannot be written explicitly. We have to refer to the previous terms to get each next term. This is called a **recursive formula**. To define a recursive sequence, we have to give as many of the first terms as we need to refer to in the recursive formula. In this last example, we had to define two terms because the recursive formula refers to the two previous terms.

e) $a_n = 2(2^{n-1})$
This is the sequence that starts with 2 and each term is two times the term before.

f) $a_n = \frac{1}{n}$
This is the sequence of fractions starting with 1 where the denominator increases by one for each term.

✏Now try the odd-numbered problems in the 9-1 Practice worksheet.