Algebra II Name_____ Date _____
Chapter 7 Summary

Major Topics

- Exponential Models
- Properties of Exponential Functions
- Logarithmic Functions
- Properties of Logarithms
- Exponential and Logarithmic Equations
- Natural Logarithms

Formulas & Notation

General form of an exponential function:

$$y = ab^x$$
$$\text{where } a \neq 0,\ b > 0,\ b \neq 1$$

Growth or Decay Models:

$$A(t) = P(1 + r)^t$$

where t is time periods, $A(t)$is the amount after t periods, P is the starting amount, and r is the rate of growth or decay ($r > 0$ growth, $r < 0$ is decay.)

Interest compounded n times per year:

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

where t is in years.

Interest compounded continuously:

$$A(t) = Pe^{rt}$$

where e is the natural base.

Logarithm change of base formula:

$$\log_b m = \frac{\log_c m}{\log_c b}$$

where b, c, m are all positive numbers and $b \neq 1$ and $c \neq 1$

Properties

For the function $y = ab^x$

- If $a > 0$ and $b > 1$ then the function is exponential growth
- If $a > 0$ and $0 < b < 1$ then the function is exponential decay
- The y-intercept is $(0, a)$
- The domain is all real numbers
- The asymptote is $y = 0$
- The range is $y > 0$

Logarithms:

$$y = b^x \equiv \log_b y = x$$
$$\log a \equiv \log_{10} a$$
$$\ln a \equiv \log_e a$$

($\equiv$ means "equivalent")

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

Vocabulary

- Exponential function
- Exponential growth
- Exponential decay
- Asymptote
- Growth factor
- Decay factor
- Natural base, e
- Compounded interest
- Compounded continuously
- Logarithm
- Common logarithm
- Natural logarithm

You should be able to:

- Determine if a function is an exponential growth or decay model
- Determine the growth or decay factor
- Use exponential models to solve word problems
- Graph an exponential function using a table of values
- Rewrite an exponential expression as an equivalent logarithmic expression
- Evaluate a logarithmic expression
- Graph a logarithmic function by using inverse exponential functions
- Use the change of base formula
- Solve exponential and logarithmic equations
- Solve natural logarithmic equations

Classwork:

1. Determine if each function is exponential growth or decay. Then graph the function. Determine the equation of the horizontal asymptote.

a) $y = 3(2)^x$ b) $f(x) = \left(\frac{1}{2}\right)^x$ c) $y = e^{-x}$

2. Determine the growth or decay factor of each function.

a) $A(t) = 1000(1 + .02)^t$ b) $A(t) = 500(1 - .17)^t$ c) $A(t) = 250\left(1 + \frac{.03}{4}\right)^{4t}$

3. You put $4500 into an account earning 3.5% interest compounded continuously. Find the amount in the account at the end of 5 years.

4. You buy a computer for business for $3000. It depreciates at a rate of 23% per year. How much is it worth in 3 years?

5. Rewrite the following statements using logarithms.

a) $3^x = y$ b) $4^{\frac{1}{2}} = 2$ c) $\left(\frac{1}{2}\right)^{-3} = 8$

6. Rewrite the following statements without logarithms, using exponents.

a) $\log_8 x = 15$ b) $\log_2 2 = 1$ c) $\log_{\frac{1}{4}} 16 = -2$

7. Evaluate the following expressions without using a calculator.

a) $\log_{10} 1000$ b) $\log_3 \frac{1}{27}$ c) $\log_9 27$ d) $\ln e$

8. Graph the following logarithmic functions by using the inverse exponential relation.

a) $f(x) = \log_3 x$ b) $f(x) = \log_2 x$

9. Rewrite the logarithms using the change of base formulas, and then use your calculator to evaluate them. Round to the hundredths place.

a) $\log_3 12$ b) $\log_{1.3} 7.18$ c) $\log_{\frac{2}{3}} 79$

10. Solve each equation. Round your answer to the nearest hundredth.

a) $2 \ln (3x - 4) = 7$ b) $e^{x+6} + 5 = 1$ c) $4^x - 5 = 12$ d) $\log (x - 25) = 2$