Algebra II
Chapter 6 Summary

Name_________________________________ Date ________

Major Topics

- Properties of Exponents
- Radicals and Roots
- Operations with Radical Expressions
- Graphing Radical Functions
- Function Operations
- Inverses

Formulas & Notation

If the n^{th} root of a is a real number, m is an integer, and $\frac{m}{n}$ is in lowest terms, then

$$a^{\frac{1}{n}} = \sqrt[n]{a} \text{ and } a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a^m}\right)$$

where if m is negative, then $a \neq 0$.

If $g(x)$ is the inverse function of $f(x)$, then $g(x) = f^{-1}(x)$

Properties

$a^0 = 1, a \neq 0$

$\frac{a^m}{a^n} = a^{m-n}$

$a^{-n} = \frac{1}{a^n}$

$(ab)^n = a^n b^n$

$(a^m)^n = a^{mn}$

$a^m \cdot a^n = a^{m+n}$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

For any real number,

$$\sqrt[n]{a^n} = \begin{cases} a \text{ if } n \text{ is odd} \\ |a| \text{ if } n \text{ is even} \end{cases}$$

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

$a\sqrt[n]{x} + b\sqrt[n]{x} = (a+b)\sqrt[n]{x}$

$a\sqrt[n]{x} - b\sqrt[n]{x} = (a-b)\sqrt[n]{x}$

Vocabulary

- n^{th} Root
- Principal root
- Radical
- Index
- Radicand
- n^{th} Power
- Extraneous solutions
- Function composition
- Inverse relation
- Inverse function

You should be able to:

- Find all real roots of a number
- Understand radical notation
- Simplify radical expressions
- Multiply and divide radical expressions
- Rationalize the denominator
- Add and subtract radical expressions
- Rewrite radicals with rational exponents
- Solve radical equations
- Add, subtract, multiply, divide and compose functions
- Find inverse relations and functions
- Graph square root and cube root functions

Classwork:

1) Find each real root.

a) $\sqrt{.36}$ b) $\sqrt[3]{-64}$ c) $-\sqrt{49}$ d) $-\sqrt[4]{16}$ e) $\sqrt{.01}$ f) $-\sqrt[3]{-8}$

2) Perform the indicated operation, if possible. Then simplify. Rationalize the denominator if necessary.

a) $\sqrt{5}\cdot\sqrt{20}$ b) $\sqrt[3]{2}\cdot\sqrt{20}$ c) $\frac{\sqrt{80}}{\sqrt{20}}$ d) $-\sqrt[3]{2}\cdot\sqrt[3]{4}$ e) $\frac{\sqrt{16x^2}}{\sqrt{48}}$ f) $\frac{\sqrt{3x^3y^4}}{\sqrt{12xy^3}}$

g) $\sqrt{5}+\sqrt{20}$ h) $\sqrt[3]{2}-\sqrt{20}$ i) $2\sqrt[3]{4}-3\sqrt[3]{32}$ j) $-\sqrt[3]{54xy}+\sqrt[3]{16xy}$

k) $(3+\sqrt{2})(3-\sqrt{2})$ l) $(\sqrt{2}+\sqrt{5})(\sqrt{2}-\sqrt{5})$

3) Rewrite the radicals as expressions with rational exponents.

a) $\sqrt{x^3}$ b) $\sqrt[4]{xy}$ c) $\sqrt{2x^5}$ d) $-\sqrt{x^3y^2}$

4) Simplify.

a) $(-5)^{\frac{1}{3}}\cdot(-5)^{\frac{1}{3}}\cdot(-5)^{\frac{1}{3}}$ b) $7^{\frac{1}{2}}\cdot 28^{\frac{1}{2}}$ c) $3^{\frac{1}{2}}\cdot 75^{\frac{1}{2}}$ d) $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ e) $(y^{\frac{2}{3}})^{-9}$

5) Solve each equation.

a) $\sqrt{4x+3}+2=5$ b) $\sqrt{2x-4}-7=-2$ c) $\sqrt{33-3x}=3$ d) $\sqrt{12-2x}+3=1$

6) Let $f(x)=2x^2+1$ and let $g(x)=2-x$. Find:

a) $f(x)\cdot g(x)$ b) $(f\circ g)(x)$ c) $2f(x)-3g(x)$ d) $f(g(-2))$

7) Find the inverse of each and then determine if it is a function. If so, write it using inverse function notation.

a) $f(x)=2x+5$ b) $f(x)=x^2-3$ c) $f(x)=x^3$ d) $f(x)=3$

8) Sketch the graphs of the functions and describe the transformations from the parent function.

a) $f(x)=\sqrt{x-1}$ b) $f(x)=\sqrt[3]{x}+2$