Name: ________________ Group Members: ________________________________

Exploration 2: Names of Functions

Date: ____________

Objective: Recall the names of certain kinds of functions.

1. $f(x) = 2x + 3$ is the equation for a **linear function.** Plot the graph and sketch the result here. Give a reason for the name *linear.*

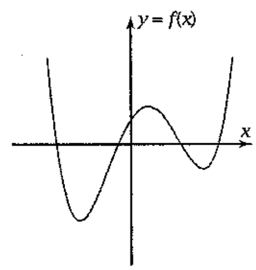
2. $f(x) = x^2 - 6x + 10$ is the equation for a **quadratic function.** Plot the graph and sketch the result. Explain how the word *quadratic* is related to the word *quadrangle.*

3. $f(x) = 3x^{0.7}$ is the equation for a **power function.** Plot the graph and sketch the result. Why do you think it is called a *power* function?

4. $f(x) = 3 \times 0.7^x$ is the equation for an **exponential function.** Plot the graph and sketch the result. How does an exponential function differ from a power function algebraically? graphically?

5. $f(x) = \frac{24}{x}$ is the equation for an **inverse variation** power function. Plot the graph for $x > 0$ and sketch the result. Why do the words "*y* varies inversely with *x*" make sense for this function? Why can the function be called a *power* function?

6. $f(x) = x^4 - 4x^3 - 43x^2 + 130x + 168$ is the equation of this **quartic function.** Why do you think the name *quartic* is used for this function? Use your grapher to find the largest value of x at which the graph crosses the x-axis.



7. $f(x) = \frac{x-4}{x-3}$ is the equation of a **rational function.** Plot the graph and sketch the result. Why do you think it is called a *rational* function? What happens to the graph at $x = 3$?

8. What did you learn as a result of doing this Exploration that you did not know before?

Name: ______________ Group Members: ______________

Exploration 1-2a: Graphs of Familiar Functions

Date: ______________

Objective: Recall the graphs of familiar functions, and tell how fast the function is changing at a particular value of *x*.

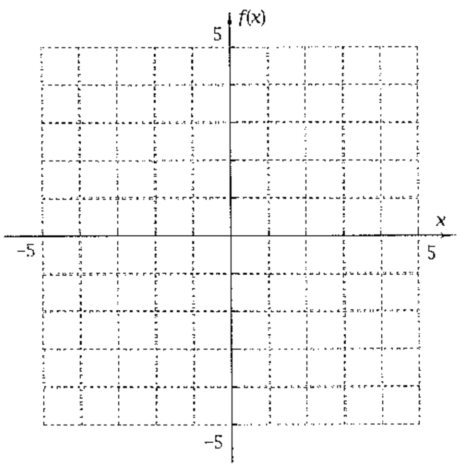
For each function:

a. Without using your grapher, sketch the graph on the axes provided.

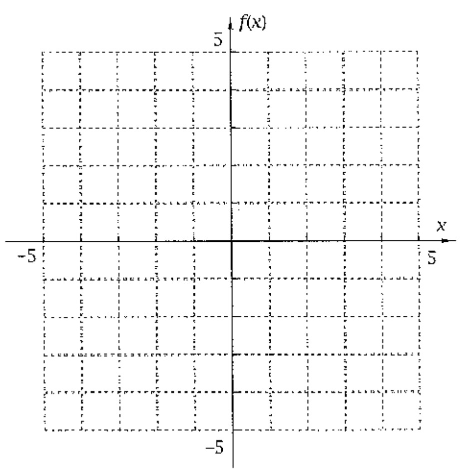
b. Confirm by grapher that your sketch is correct.

c. Tell whether the function is increasing, decreasing, or not changing when $x = 1$. If it is increasing or decreasing, tell whether the rate of change is slow or fast.

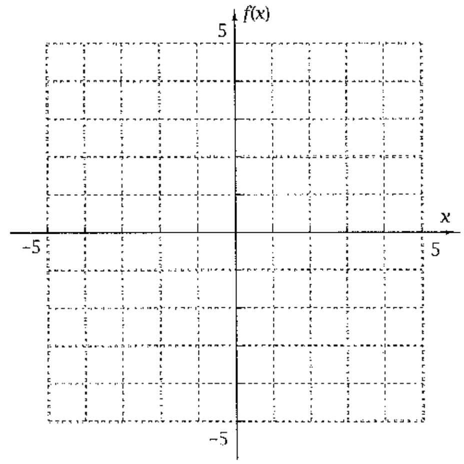
1. $f(x) = 3^{-x}$



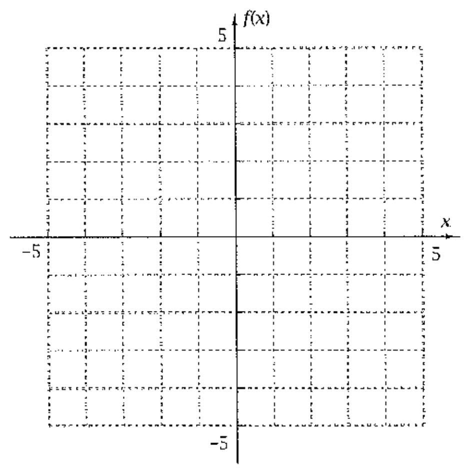
2. $f(x) = \sin \frac{\pi}{2}x$



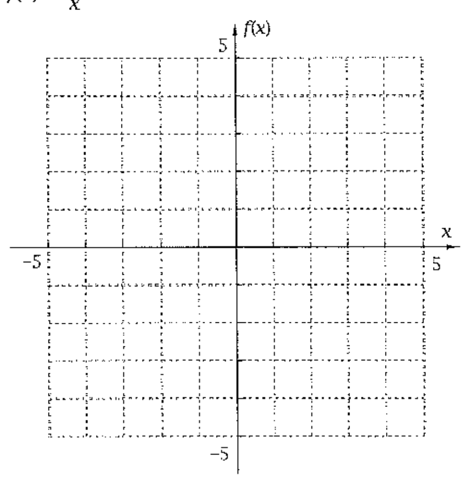
3. $f(x) = x^2 + 2x - 2$



4. $f(x) = \sec x$



5. $f(x) = \frac{1}{x}$



1-2 Rate of Change by Equation, Graph, or Table

Types of Functions

Linear

$f(x) = mx + b$; m and b stand for constants, $m \neq 0$

Quadratic

$f(x) = ax^2 + bx + c$; a,b and c stand for constants, $a \neq 0$

Polynomial

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots + a_nx^n$;

$a_0, a_1, \ldots$ stand for constants, n is a positive integer,

$a_n \neq 0$ (nth degree polynomial function)

Power

$f(x) = ax^n$; a and n stand for constants

Exponential

$f(x) = ab^x$; a and b stand for constants , $a \neq 0, b > 0, b \neq 1$

Rational Algebraic:

$$f(x) = \frac{polynomial}{polynomial}$$

Absolute value

$f(x)$ contains $|\text{variable expression}|$

Trigonometric or Circular

$f(x)$ contains $\cos x, \sin x, \tan x, \cot x, \sec x, \csc x$

2-1Numerical Approach to the Definition of Limit

Informal Definition: The limit of a function *f* as x approaches c is the y-value that *f(x)* stays close to when x is kept close enough to c but not equal to c.

Exploratory Problem Set 2-1 p. 33

Exploration 2.1 a

Name: ____________________ Group Members: ____________________

Exploration 2-1a: Introduction to Limits

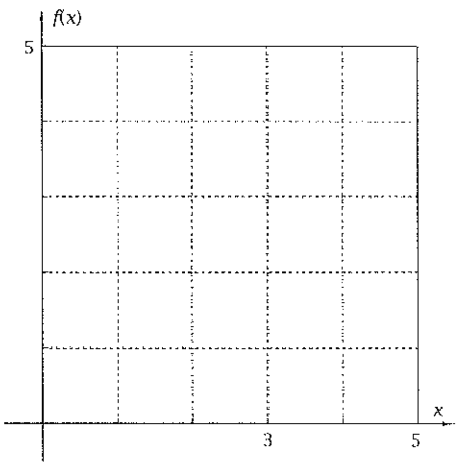
Date: ____________

Objective: Find the limit of a function that approaches an indeterminate form at a particular value of x and relate it to the definition.

1. Plot on your grapher the graph of this function.

$$f(x) = \frac{x^3 - 7x^2 + 17x - 15}{x - 3}$$

Use a friendly window with $x = 3$ as a grid point, but with the grid turned off. Sketch the results here. Show the behavior of the function in a neighborhood of $x = 3$.



2. Substitute 3 for x in the equation for $f(x)$. What form does the answer take? What name is given to an expression of this form?

3. The graph of f has a **removable discontinuity** at $x = 3$. The y-value at this discontinuity is the **limit** of $f(x)$ as x approaches 3. What number does this limit equal?

4. Make a table of values of $f(x)$ for each 0.1 unit change in x-value from 2.5 through 3.5.

x	$f(x)$
2.5	
2.6	
2.7	
2.8	
2.9	
3.0	
3.1	
3.2	
3.3	
3.4	
3.5	

5. Between what two numbers does $f(x)$ stay when x is kept in the open interval (2.5, 3.5)?

6. Simplify the fraction for $f(x)$. Solve numerically to find the two numbers close to 3 between which x must be kept if $f(x)$ is to stay between 1.99 and 2.01.

7. How far from $x = 3$ (to the left and to the right) are the two x-values in Problem 6?

8. For the statement "If x is within ________ units of 3 (but not equal to 3), then $f(x)$ is within 0.01 unit of 2," write the largest number that can go in the blank.

9. The formal definition of limit is

$$L = \lim_{x \to c} f(x) \text{ if and only if}$$

- for any positive number ε (no matter how small)
- there is a positive number δ such that
- if x is within δ units of c, but not equal to c,
- then $f(x)$ is within ε units of L.

The four numbers L, c, ε, and δ all appear in Problem 8. Which is which?

10. What did you learn as a result of doing this Exploration that you did not know before?

2-3 The Limit Theorems

WARMUP:

Divide $\dfrac{x^3 - x^2 - 2x - 12}{x - 3}$

Activity:
Enter the following functions into your calculator.

$f(x) = x^2$

$g(x) = x + 1$

$h(x) = f(x) + g(x)$

$k(x) = f(x) \cdot g(x)$

$r(x) = f(x) / g(x)$

Examine the limit as x approaches 3 of each function.

What conjectures can you make?

The limit of a product of two functions is the product of the two limits.

The limit of a sum of two functions is the sum of the two limits.

The limit of a quotient of two functions is the quotient of the two limits, provided that the denominator does not approach zero.

Ex. 1 Evaluate $\lim_{x \to 3} x$

The limit of x as x approaches c is simply c.

Ex. 2 Evaluate $\lim_{x \to 2} 5x$

The limit of a constant times a function equals the constant times the limit.

Ex. 3 Evaluate $\lim_{x \to 2} 4$

The limit of a constant is that constant.

THE LIMIT THEOREMS are listed in your book on page 42!

Ex. 4 Evaluate $\lim_{x \to 3} \frac{x^3 - x^2 - 2x - 12}{x - 3}$

Ex. 5 Evaluate $\lim_{x \to 5} \frac{5x^2 - 125}{x - 5}$

Ex. 6 Evaluate $\lim_{x \to 3} \frac{x^3 + x^2 - 5x - 21}{x - 3}$

Ex. 7 Evaluate $\lim_{x \to 0} \frac{\sin(x)}{x}$

Name: ____________________ Date: ____________________

Period: _______ Calculus Honors 2.3 Extra Practice

USE THE LIMIT THEOREMS TO FIND THE LIMIT, IF IT EXISTS.

1. $\lim_{x\to\sqrt{2}} 15$

2. $\lim_{x\to -2} x$

3. $\lim_{x\to 4}(3x-4)$

4. $\lim_{x\to -2}\dfrac{x-5}{4x+3}$

5. $\lim_{x\to 1}(-2x+5)^4$

6. $\lim_{x\to 0}\dfrac{4-\sqrt{16+x}}{x}$

7. $\lim_{x\to 1}\dfrac{x^2-1}{x-1}$

8. $\lim_{x\to 0}\dfrac{\sqrt{x^2+9}-3}{x^2}$

9. $\lim_{x\to 2}\dfrac{x^2+x-6}{x-2}$

10. $\lim_{x\to 0}\dfrac{(4+x)^2-16}{x}$

11. $\lim_{x\to 1}\dfrac{x^3-1}{x^2-1}$

12. $\lim_{x\to 4}\dfrac{x^2-4x}{x^2-3x-4}$

13. $\lim_{x\to 0}\dfrac{(2+x)^2-4}{x}$

14. $\lim_{x\to 2}\dfrac{x^4-16}{x-2}$

15. $\lim_{x\to 1}\dfrac{x^2-1}{x-1}$

16. $\lim_{x\to -2}\frac{x^3+2x^2-1}{5-3x}$

17. $\lim_{x\to 1} g(x)$ $\quad g(x)=\begin{cases} x+1 & \text{if } x\neq 1 \\ \pi & \text{if } x=1 \end{cases}$

18. $\lim_{h\to 0}\frac{(3+h)^2-9}{h}$

19. $\lim_{t\to 0}\frac{\sqrt{t^2+9}\;-3}{t^2}$

20. $\lim_{x\to 0}|x|$

21. $\lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7}$

22. $\lim_{x\to -4}\frac{\frac{1}{4}+\frac{1}{x}}{4+x}$

23. $\lim_{x\to 0}\frac{(4+x)^2-16}{x}$

24. $\lim_{t\to 2}\frac{t^2-3t+2}{t^2-4}$

25. $\lim_{x\to 0}\frac{\frac{1}{2+x}-\frac{1}{2}}{x}$

26. $\lim_{x\to 0}\frac{x+\sin x}{x}$

27. Given $\lim_{x\to a} f(x)=-3$; $\lim_{x\to a} g(x)=0$; $\lim_{x\to a} h(x)=8$, find

a. $\lim_{x\to a}[f(x)+h(x)]$

b. $\lim_{x\to a}[f(x)]^2$

c. $\lim_{x\to a}\sqrt[3]{h(x)}$

d. $\lim_{x\to a}\frac{f(x)}{g(x)}$

Name: ______________________________ Date: ____________________

Calculus Honors: More Practice with Limits

Evaluate the limits (if they exist). Show the algebraic steps and then confirm on your graphing calculator.

1. $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

2. $\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 - 9}$

3. $\lim_{x \to 4} \frac{\sqrt{x+5} - 3}{x - 4}$

4. $\lim_{x \to 5} \frac{x^3 - 4x^2 - 6x + 5}{x - 5}$

5. $\lim_{x \to 1} \frac{x - 3}{x^2 + 4}$

6. $\lim_{x \to 0} \frac{(4+x)^2 - 16}{x}$

7. $\lim_{x \to -1} g(x)$ for $g(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

8. $\lim_{x \to 0} g(x)$ for $g(x) = \begin{cases} x^2 + 2 & \text{if } x \geq -1 \\ 2 - x & \text{if } x < -1 \end{cases}$

9. $\lim_{x \to \pi} h(x)$ for $h(x) = \begin{cases} \sin x & \text{if } x \geq \frac{\pi}{2} \\ \cos x & \text{if } x < \frac{\pi}{2} \end{cases}$

10. $\lim_{x \to 5} f(x)$ for $f(x) = \begin{cases} x^3 - 8 & \text{if } x \geq 0 \\ x - 2 & \text{if } x < 0 \end{cases}$

2-4 Continuity and Discontinuity

WARMUP: Where is the following function discontinuous? $f(x)=\dfrac{x^2-9}{x-3}$

Types of discontinuity:

1. Removable (hole) (come from the domain restrictions that can be factored out)

2. Step or (jump)

3. Infinite (vertical asymptotes)

Find the points of discontinuity of the function and identify the type.

1. $y = \dfrac{1}{(x-2)^2}$

2. $y = \dfrac{x-1}{x^2 - 4x + 3}$

Limits:

$\lim_{x \to c^-} f(x)$ x → c from the left

(through values of x on the negative side of c)

$\lim_{x \to c^+} f(x)$ x → c from the right

(through values of x on the positive side of c)

L= $\lim_{x \to c} f(x)$ if and only if

L= $\lim_{x \to c^-} f(x)$ and $\lim_{x \to c^+} f(x)$

Definition of Continuity (at a point):

A function is continuous at *c* iff:

1. $f(c)$ exists

2. $\lim_{x \to c} f(x)$ exists

3. $\lim_{x \to c} f(x) = f(c)$

Continuity on an INTERVAL: A function is continuous on an interval of x-values iff it is continuous at each value of x in that interval. At the end points of a closed interval, only the one-sided limits need to equal the function value.

A CUSP is a point on the graph at which the function is continuous but the derivative is discontinuous.

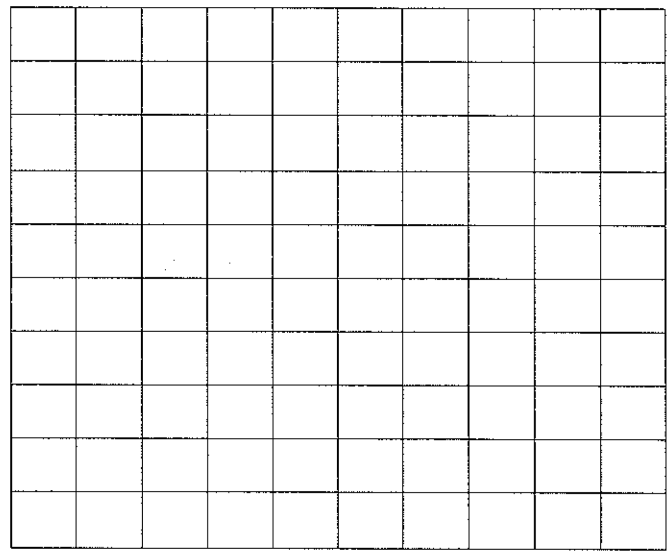
A cusp is a sharp point or an abrupt change in direction.

One-Sided Limits

Each part of a piecewise function is called a BRANCH.

Example:

Graph:



$$f(x)=\begin{cases} x^2-1 & -1\le x<0 \\ 2x & 0<x<1 \\ 1 & x=1 \\ -2x+4 & 1<x<2 \\ 0 & 2<x<3 \end{cases}$$

1.
   a. Does f(-1) exist?
   b. Does $\lim_{x\to -1^+} f(x)$ exist?
   c. Does $\lim_{x\to -1^+} f(x)=f(-1)$?

2.
   a. Does f(1) exist?
   b. Does $\lim_{x\to 1} f(x)$ exist?
   c. Does $\lim_{x\to 1} f(x)=f(1)$?

3.
   a. Does f(2) exist?
   b. Does $\lim_{x\to 2^+} f(x)$ exist?
   c. Does $\lim_{x\to 2^+} f(x)=f(2)$?

4. For what values of x is the function continuous?

5. Could you redefine the function to make it continuous for all $-1\le x\le 3$? If not, at what points can you make it continuous?

Examples. Find a value for a so that the given function is continuous.

1. $f(x)=\begin{cases} x^2-1 & x<3 \\ 2ax & x\geq 3 \end{cases}$

2. $f(x)=\begin{cases} 2x+3 & x\leq 2 \\ ax+1 & x>2 \end{cases}$

3. $f(x)=\begin{cases} x^2+x+a & x<1 \\ x^3 & x\geq 1 \end{cases}$

Name: ______________________________ Date: ______________________________
Period: ________ Calculus Honors

1. Graph the following piecewise function.
Identify True or False for parts a-h.

$$f(x)=\begin{cases} -(x+1)^2+1 & -1\le x<0 \\ 2x^2 & 0\le x<1 \\ 1 & 1\le x<2 \\ 2 & x=2 \\ 1 & 2<x\le 3 \end{cases}$$

a. True or False? $\lim_{x\to -1^+} f(x)=1$

b. True or False? $\lim_{x\to 2} f(x)$ does not exist

c. True or False? $\lim_{x\to 2} f(x)=2$

d. True or False? $\lim_{x\to 1^-} f(x)=2$

e. True or False? $\lim_{x\to 1^+} f(x)=1$

f. True or False? $\lim_{x\to 1} f(x)$ does not exist

g. True or False? $\lim_{x\to c} f(x)$ exists at every c in (-1,1)

h. True or False? $\lim_{x\to c} f(x)$ exists at every c in (1,3)

i. Is $f(x)$ continuous at $x=1$? Justify your answer.

j. Is $f(x)$ continuous at $x=2$? Justify your answer.

2. Find a value for a so that the given function is continuous.

$$f(x)=\begin{cases} 5-x^2, & \text{if } x<1 \\ 2ax-1, & \text{if } x\ge 1 \end{cases}$$

Name: ______________________ Group Members: ______________________

Exploration 2-4a: Continuous and Discontinuous Functions

Date: ____________

Objective: Given a function specified by two different rules, make the function continuous at the boundary between the two branches.

Let f be the piecewise function defined by

$$f(x) = \begin{cases} x + 1, & \text{if } x < 2 \\ k(x - 5)^2, & \text{if } x \geq 2 \end{cases}$$

where k stands for a constant.

1. Plot the graph of f for $k = 1$. Sketch the result.

2. Function f is **discontinuous** at $x = 2$. Tell what it means for a function to be discontinuous.

3. Find $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$. (The second limit will be in terms of k.) What must be true of these two limits for f to be **continuous** at $x = 2$?

4. Find the value of k that makes f continuous at $x = 2$. Sketch the graph of f for this value of k.

5. The graph in Problem 4 has a **cusp** at $x = 2$. What is the origin of the word *cusp*, and why is it appropriate to use in this context?

6. Suppose someone asks, "Is $f(x)$ increasing or decreasing at $x = 2$ with k as in Problem 4?" How would you have to answer that question? What, then, can you conclude about the derivative of a function at a point where the graph has a cusp?

7. What did you learn as a result of doing this Exploration that you did not know before?

Name: ______________________________ Date: ______________________________
Period: _____ Calculus Honors

∞ 2.5 *Limits Involving Infinity* ∞

1. Use your calculator to examine the function $f(x)=\frac{2x+2}{x-2}$.

   a. Find $\lim_{x\to\infty} f(x)$. (ie. what value does the function approach as x gets really REALLY big?)

   b. Does $\lim_{x\to 2} f(x)$ exist? Why or why not?

2. Use your calculator to examine the function $f(x)=\frac{x^3+2x+3}{4x^3-4}$.

   a. Find $\lim_{x\to\infty} f(x)$.

   b. Does $\lim_{x\to 1} f(x)$ exist? Why or why not?

Limits involving infinity can be one of two types:

a. ____

b. ____

3. Use your calculator to find $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ for :

a. $f(x) = \dfrac{x}{\sqrt{x^2+1}}$

b. $f(x) = \dfrac{e^{-x}}{x}$

4. Find the vertical asymptotes of the graph. Describe the behavior of the function to the left and right of each vertical asymptote.

a. $f(x) = \dfrac{1}{x^2-4}$

b. $f(x) = \dfrac{1-x}{2x^2-5x-3}$

c. $f(x) = \dfrac{x^2-2x}{x+1}$

Name: ______________ Group Members: ______________

Exploration 2-5a: Limit As x Approaches Infinity

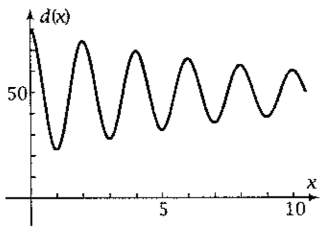
Date: ______________

Objective: Discover what it means for a function to approach a limit as x approaches infinity.

A pendulum is pulled away from its rest position and let go. As it swings back and forth, its distance, $d(x)$, in cm from the wall, is given by the equation

$$d(x) = 50 + 30(0.9)^x \cdot \cos \pi x$$

where x is time in seconds since it was let go. The graph of function d shows that friction decreases the amplitude of the swings as time goes on.



1. Plot $y_1 = d(x)$ on your grapher using the window shown. Does the graph agree with the figure?
2. The number 50 is the **limit of $d(x)$ as x approaches infinity.** Plot lines $y_2 = 52$ and $y_3 = 48$. If x is large enough, the graph stays within these two lines. By experimenting with the window, find the smallest possible number D for which $d(x)$ stays within 2 units of 50 for *all* values of $x > D$.
3. The quantity $30(0.9)^x$ in the equation for $d(x)$ is the **amplitude** of the cosine function. Plot $y_4 = 50 + 30(0.9)^x$. Is this graph really an upper bound for $d(x)$?
4. Find a new value of $x = D$ that makes $30(0.9)^D = 2$. Change the window so that x goes from about 5 below this value to about 5 above. Use a y-range of [45, 55]. Sketch the result. Does $d(x)$ really stay within 2 units of 50 for all $x > D$ for this value of D?
5. Find quickly a value of $D > 0$ such that $d(x)$ stays within 0.1 unit of 50 whenever $x > D$. Show how you get your answer.

Because infinity is not a number, you cannot make x close to infinity. So the limit as x approaches infinity must be modified to say that x is kept far enough away from zero. Here is a formal definition.

$L = \lim_{x \to \infty} f(x)$ if and only if

- for any $\varepsilon > 0$ (no matter how small)
- there is a number $D > 0$ such that
- if $x > D$,
- then $f(x)$ is within ε units of L.

6. For the function d in this Exploration, explain why this statement is false: "The larger x gets, the closer $d(x)$ gets to 50." Explain how the words "stays within" in the definition of limit avoid the misconception that $d(x)$ is always getting closer to the limit.
7. Tell the real-world meaning of the limit 50 for the function d in this Exploration.
8. What did you learn as a result of doing this Exploration that you did not know before?

2-6 The Intermediate Value Theorem and its Consequences

Property: The Intermediate Value Theorem

If the function f is continuous for all x in the closed interval [a,b], and y is a number between $f(a)$ and $f(b)$, then there is a number x = c in (a,b) for which $f(c) = y$.

"If you pick a value of y between any two values of $f(x)$, there is an x value in the domain that gives exactly that y values for $f(x)$."

Example:
If $f(x) = x^3 - 4x^2 + 2x + 7$, use the intermediate value theorem to conclude that a value of x = c occurs between 1 and 3 for which $f(c)$ is exactly equal to 5.

Name: ______________________________ Date: ______________________________

Period: ______ Calculus Honors 2.6 Extra Practice

VERIFY THAT THE INTERMEDIATE VALUE THEOREM APPLIES TO THE INDICATED INTERVAL AND FIND THE VALUE OF C GUARANTEED BY THE THEOREM.

1. $f(x)=x^2+x-1$ $[0,5],\ f(c)=11$

2. $f(x)=x^2-6x+8$ $[0,3],\ f(c)=0$

3. $f(x)=x^3-x^2+x-2$ $[0,3],\ f(c)=4$

4. $f(x)=\dfrac{x^2+x}{x-1}$ $\left[\dfrac{5}{2},4\right], f(c)=6$