

3/23/15: AP Stat – Summary Notes for Inference Procedures

An Overview Summary of Inference Procedures for:

Ch. 8 \Rightarrow Estimating with Confidence (Confidence Intervals)

Ch. 9 \Rightarrow Testing A Claim (Significance Tests)

Ch. 10 \Rightarrow Comparing Two Populations or Groups

Ch. 11 \Rightarrow Inference for Distributions of Categorical Data

Ch. 12 \Rightarrow More About Regression

What Stays The Same

- Formula for Confidence Interval (CI) is: estimate \pm margin of error
which, in detail, is: statistic \pm critical value \cdot standard deviation of the statistic
- Type I, Type II errors, α , β , Power, and p-value concepts
- Four Step Procedure:
State: State the question, the parameter(s) in words and symbols, and the hypotheses and alpha if it is a significance test
Plan: Indicate Procedure and Check Conditions
Do: Perform Calculations
Conclude: Give the statistical result and interpretation in context
- We will always reject the null hypothesis when p-value $<$ α
- A 1- α % Confidence Interval for means will still correspond exactly to a two-sided hypothesis test for means with significance level α .

What Differs For Different Procedures

- Conditions are met differently
- The test statistics used have different distributions
- Procedures (calculations) are different for each test
- Interpretations in context depend on each scenario

\Rightarrow Know how to tell what type of inference is required. This takes practice!!!

Some Ch. 10 Summary Notes

Inference for Two Populations:

Two Sample Setting

- ⇒ Compare response to two treatments or characteristics of two populations.
- ⇒ Separate sample from each treatment or population.
- ⇒ Responses of each group are independent of those in the other group.

We do NOT match observations (no pairing) in a two sample setting, and samples can be of different sizes.

In formulas, we subscript all of the variables to identify which population they represent:

<u>Variable</u>	<u>Symbol</u>	<u>Population 1</u>	<u>Population 2</u>
Population mean	μ	μ_1	μ_2
Population variance	σ^2	σ_1^2	σ_2^2
Population standard deviation	σ	σ_1	σ_2
Population proportion	p	p_1	p_2
Population size	N	N_1	N_2
Sample mean	\bar{x}	\bar{x}_1	\bar{x}_2
Sample variance	s^2	s_1^2	s_2^2
Sample standard deviation	s	s_1	s_2
Sample proportion	\hat{p}	\hat{p}_1	\hat{p}_2
Sample size	n	n_1	n_2

Comparing Two Population Means (or Two Treatment Means)

Comparing two means becomes a difference.

We can either do a CI for the difference of the means, or we can do a significance test for comparing two means, which really becomes a test of the difference of the means.

I. Significance Test for comparing Two Population means:

State: Hypotheses

$$H_0: \mu_1 = \mu_2 \quad \text{becomes} \quad H_0: \mu_1 - \mu_2 = 0$$

$$H_a: \mu_1 > \mu_2 \quad \text{becomes} \quad H_a: \mu_1 - \mu_2 > 0$$

$$H_a: \mu_1 \neq \mu_2 \quad \text{becomes} \quad H_a: \mu_1 - \mu_2 \neq 0$$

$$H_a: \mu_1 < \mu_2 \quad \text{becomes} \quad H_a: \mu_1 - \mu_2 < 0$$

Plan: Procedure & Conditions

⇒ Use a 2-sample-t procedure, assuming population standard deviations are not known

- Random: Need two SRS's from two populations (or two distinct treatments)
- Independence: Samples must be independent of each other as well as within their own populations. **PAIRED DATA FAILS TO BE INDEPENDENT!**

Observations on the same individuals are NOT independent. Randomized treatment of experimental subjects satisfies this condition.

When sampling without replacement, check that $n_1 \leq \frac{1}{10} N_1$ and $n_2 \leq \frac{1}{10} N_2$.

- Normal/Large Counts: Both population distributions (or the true distributions of responses to the two treatments) are Normal or both sample sizes are large ($n_1 \geq 30$ and $n_2 \geq 30$). If either population (treatment) distribution has unknown shape and the corresponding sample size is less than 30, use a graph of the sample data to assess the Normality of the population (treatment) distribution. Do not use two-sample t procedures if the graph shows strong skewness or outliers.

Do: Calculations

The Test statistic is t, and the sample statistic is the difference of the two sample means. We always assume the null hypothesis is true when calculating the test statistics

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t \text{ (d.f.)}$$

Degrees of freedom (d.f.) is complicated:

use calculator d.f. or conservative calculation: lesser of $n_1 - 1, n_2 - 1$

Note: The calculator can produce a d.f. that is larger than either of the two sample sizes. This will be more accurate. Note that the *conservative* d.f. will correspond to a distribution with *thicker tail areas*.

The p-value is calculated the same way as in a one-sample test. Use the t distribution for the test statistic in tcdf (or the t-table) to calculate the area beyond the test-statistic. Remember to double this value for a two-tailed test.

Conclude:

Interpret in context the same as a one-sample test. If the p-value is less than alpha, reject the null. Put your conclusion in the context of the problem. Which hypothesis is supported by the sample data?

II. Confidence Interval (CI)

The conditions are the same as the conditions for a significance test.

Recall the general format for a confidence interval is:

point estimate \pm margin of error

On your formula sheet, this is given in a more specific form:

statistic \pm critical value \cdot std dev of statistic

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where t^* is the critical value associated with the confidence level. If using the t-table, choose the column based on the desired confidence interval from the bottom row of the table. Degrees of freedom works the same as significance test: you can use the calculator value, or for a more conservative (wider) interval, choose the lesser of $n_1 - 1, n_2 - 1$.

Note: NEVER use POOLED. (It is always right to NOT use it, but sometimes wrong to use it. Just avoid it so as not to memorize a bunch of unnecessary conditions.)

Be sure to interpret your confidence interval in context: "We are <CI>% confident that the interval from <CI Lower> to <CI Upper> captures the true difference in <describe the population means in context>."