

Evaluate the following functions for the given values of  $x$ :

1.  $f(x) = 2x + 3$  ,  $x = -3$

2.  $f(x) = -2x^2 + 1$  ,  $x = -2$

3.  $g(x) = x^3 + x^2 + x + 1$  ,  $x = -1$

4.  $h(x) = |4 - x|$  ,  $x = 7$

5.  $f(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$  ,  $x = 4$  (Hint: Use the remainder theorem! 😊 )

## Algebra 2: Vocabulary of Functions

1.	algebraic expression	a number or idea represented using numbers, letters, grouping symbols, and/or operations
2.	domain	the set of all input values of a relation
3.	evaluate	find the value of an expression
4.	function	a relation that has only one output for each input in the domain
5.	function definition	a statement which associates the function name and its input variable with an output rule
6.	function notation	$y = f(x)$ Instead of the variable $y$ we write $f(x)$ and we say "f of x".
7.	graph	representation of a relation using input values as the horizontal coordinates and output values as the vertical coordinates
8.	input	the $x$ coordinate of an ordered pair
9.	inverse	a relation in which all the ordered pairs are reversed: the outputs become the inputs, and the inputs become the outputs
10.	mapping diagram	represents a relation using arrows to connect the input values to the corresponding output values
11.	ordered pair	a pair of values $(x, y)$ where $x$ is the input value and $y$ is the output value
12.	output	the $y$ coordinate of an ordered pair
13.	range	the set of all output values of a relation
14.	relation	a rule that associates input values with output values to create ordered pairs
15.	table of values	an organized method of listing ordered pairs in rows and columns
16.	vertical line test	a graphical way to determine if a relation is a function: if it is a function, then there is no place on the graph that one vertical line can pass through two points
17.	zero of a function	the $x$ value(s) that correspond to a $y$ -value of zero; real zeros are the $x$ -intercepts of the graph

## Algebra 2: Operations with Functions

### Operations with Functions

There are **five** operations that can be performed with functions.

1. Addition
2. Subtraction
3. Multiplication
4. Division
5. Composition

To **add, subtract, multiply or divide** functions, substitute the function definition for the function and perform the algebraic operations indicated.

Examples:

$$f(x) = 3x + 2$$

Given the following functions:

$$g(x) = 9 - 2x$$

$$h(x) = -x - 1$$

$$k(x) = -1$$

Perform the indicated operations:

a)  $(f + g)(x) = f(x) + g(x)$

b)  $(g - h)(x) = g(x) - h(x)$

c)  $(f \cdot h)(x) = f(x) \cdot h(x)$

d)  $\left(\frac{h}{k}\right)(x) = \frac{h(x)}{k(x)}$

## Algebra 2: Operations with Functions

To find the **domain** of the new function when you operate with functions, you must choose the set of all  $x$  values from the previous domains that can be evaluated in the new expression.

This means, you can only include  $x$ -values that could go into each original function, and then exclude any of those that can NOT go into the new function.

Note: When there are no exclusions, the domain of  $x$  is all real numbers.

Find the domain of  $f(x)$ ,  $g(x)$ , and then the domain of the resulting expression for the indicated operation:

1.  $f(x) = x^2 + 1$ ,  $g(x) = 3 - x$ ,  $(f - g)(x)$

2.  $f(x) = |x|$ ,  $g(x) = -2x + 5$ ,  $(f + g)(x)$

3.  $f(x) = 3$ ,  $g(x) = \sqrt{x}$ ,  $(f \cdot g)(x)$

4.  $f(x) = 14x - 21$ ,  $g(x) = 7x$ ,  $\left(\frac{f}{g}\right)(x)$

## Algebra 2: Operations with Functions

The **composition of functions** is when we **substitute an entire function for the x (input) in another function.**

**The notation** for composition is an open circle:

$$f(x) \circ g(x) \text{ and we say, "f composed with g of x".}$$

We can also write:  $(f \circ g)(x)$  and we say "**f of g of x.**"

We can also write:  $f(g(x))$  and we say the same "**f of g of x.**"

Example:

$$f(x) = 3x + 2$$

Given the following functions:  $g(x) = 9 - 2x$

$$h(x) = -x - 1$$

Perform the indicated operations:

e) f composed with g means, substitute g into f.

$$\begin{aligned} f(x) \circ g(x) &= f(9 - 2x) \\ &= 3(9 - 2x) + 2 \\ &= 27 - 6x + 2 \\ &= 29 - 6x \end{aligned}$$

f) g composed with f means, substitute f into g.

$$\begin{aligned} g(x) \circ f(x) &= g(3x + 2) \\ &= 9 - 2(3x + 2) \\ &= 9 - 6x - 4 \\ &= 5 - 6x \end{aligned}$$

g) g of h of x means, substitute h into g.

$$\begin{aligned} (g \circ h)(x) &= g(-x - 1) \\ &= 9 - 2(-x - 1) \\ &= 9 + 2x + 2 \\ &= 11 + 2x \end{aligned}$$

h) h of g of x means, substitute g into h.

$$\begin{aligned} h(g(x)) &= h(9 - 2x) \\ &= -(9 - 2x) - 1 \\ &= -9 + 2x - 1 \\ &= -10 + 2x \end{aligned}$$

## 6-6

## Practice

Form G

Let  $f(x) = 4x - 1$  and  $g(x) = 2x^2 + 3$ . Perform each function operation and then find the domain.

1.  $(f + g)(x)$

2.  $(f - g)(x)$

3.  $(g - f)(x)$

4.  $(f \cdot g)(x)$

5.  $\frac{f}{g}(x)$

6.  $\frac{g}{f}(x)$

Let  $f(x) = 2x$  and  $g(x) = \sqrt{x}$ . Perform each function operation and then find the domain of the result.

7.  $(f + g)(x)$

8.  $(f - g)(x)$

9.  $(g - f)(x)$

10.  $(f \cdot g)(x)$

11.  $\frac{f}{g}(x)$

12.  $\frac{g}{f}(x)$

Let  $f(x) = -3x + 2$ ,  $g(x) = \frac{x}{5}$ ,  $h(x) = -2x^2 + 9$ , and  $j(x) = 5 - x$ . Find each value or expression.

13.  $(f \circ j)(3)$

14.  $(j \circ h)(-1)$

15.  $(h \circ g)(-5)$

16.  $(g \circ f)(a)$

17.  $f(x) + j(x)$

18.  $f(x) - h(x)$

19.  $(g \circ f)(-5)$

20.  $(f \circ g)(-2)$

21.  $3f(x) + 5g(x)$

22.  $g(f(2))$

23.  $g(f(x))$

24.  $f(g(1))$

25. A video game store adds a 25% markup on each of the games that it sells. In addition to the manufacturer's cost, the store also pays a \$1.50 shipping charge on each game.

a. Write a function to represent the price  $f(x)$  per video game after the store's markup.

b. Write a function  $g(x)$  to represent the manufacturer's cost plus the shipping charge.

c. Suppose the manufacturer's cost for a video game is \$13. Use a composite function to find the cost at the store if the markup is applied after the shipping charge is added.

d. Suppose the manufacturer's cost for a video game is \$13. Use a composite function to find the cost at the store if the markup is applied before the shipping charge is added.

## 6-6

## Practice (continued)

Form G

26. The formula  $V = \frac{4}{3}\pi r^3$  expresses the relationship between the volume  $V$  and radius  $r$  of a sphere. A weather balloon is being inflated so that the radius is changing with respect to time according to the equation  $r=t+1$ , where  $t$  is the time, in minutes, and  $r$  is the radius, in feet.
- Write a composite function  $f(t)$  to represent the volume of the weather balloon after  $t$  minutes. Do not expand the expression.
  - Find the volume of the balloon after 5 minutes. Round the answer to two decimal places. Use 3.14 for  $\pi$ .
27. A boutique prices merchandise by adding 80% to its cost. It later decreases by 25% the price of items that do not sell quickly.
- Write a function  $f(x)$  to represent the price after the 80% markup.
  - Write a function  $g(x)$  to represent the price after the 25% markdown.
  - Use a composition function to find the price of an item, after both price adjustments, that originally costs the boutique \$150.
  - Does the order in which the adjustments are applied make a difference? Explain.
28. A department store has marked down its merchandise by 25%. It later decreases by \$5 the price of items that have not sold.
- Write a function  $f(x)$  to represent the price after the 25% markdown.
  - Write a function  $g(x)$  to represent the price after the \$5 markdown.
  - Use a composition function to find the price of a \$50 item after both price adjustments.
  - Does the order in which the adjustments are applied make a difference? Explain.

Let  $g(x) = x^2 - 5$  and  $h(x) = 3x + 2$ . Perform each function operation.

29.  $(h \circ g)(x)$

30.  $g(x) \cdot h(x)$

31.  $-2g(x) + h(x)$