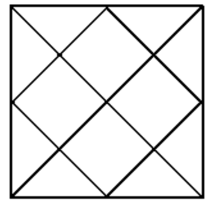
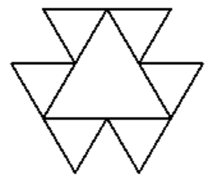
Algebra 2 – Warm Up – Mar. 11, 2014 Name: ___________________________________

1. A floor tile is made up of smaller squares.
Each square measures 3 in. on each side.
Find the perimeter of the floor tile.



2. A section of mosaic tile wall has the design shown at the right.
The design is made up of equilateral triangles.
Each side of the large triangle is 4 in. and
each side of a small triangle is 2 in.
Find the total area of the design to the nearest tenth of an inch.



Rational Exponents and Radicals (nth Roots)

Notation:

There are two ways to write an expression for the $n^{th}$ root of a number.

We already learned to use the **radical** notation: $\sqrt[3]{27}$ means "the 3rd root of 27".

Another way to express this idea is to use **rational exponents**. Recall that a rational number can be written as a fraction made of two integers. So a rational exponent is just an exponent in fraction form.

The exponent for the $n^{th}$ root of a number is $\frac{1}{n}$.

Example: $\sqrt[3]{27}$ can be written as $(27)^{\frac{1}{3}}$ and in both cases they represent the number 3.

When we use rational exponents, the **denominator** represents the **root** and the **numerator** represents the **power**.

Examples:

a) $\sqrt[3]{x^2} = x^{\frac{2}{3}}$ b) $\frac{1}{\sqrt[4]{x}} = x^{-\frac{1}{4}}$ c) $81^{\frac{1}{2}} = \sqrt{81}$ d) $16^{-\frac{3}{2}} = \frac{1}{\sqrt{16^3}}$ e) $(-64)^{\frac{1}{3}} = \sqrt[3]{-64}$

You can rewrite expressions from one form to the other to make simplifying easier. Rational exponents follow the same properties as integer exponents, so that is a convenient way to multiply radicals that have different roots.

Examples:

What is the simplified form of each expression?

**a.** $36^{\frac{1}{4}} \cdot 36^{\frac{1}{4}}$

$36^{\frac{1}{4}} \cdot 36^{\frac{1}{4}} = 36^{\frac{1}{4}+\frac{1}{4}}$ Use $a^m \cdot a^n = a^{m+n}$.

$= 36^{\frac{1}{2}}$ Add.

$= \sqrt[2]{36}$ Use $x^{\frac{1}{n}} = \sqrt[n]{x}$.

$= 6$ Simplify.

**b.** Write $(6x^{\frac{2}{3}})(2x^{\frac{3}{4}})$ in simplified form.

$(6x^{\frac{2}{3}})(2x^{\frac{3}{4}}) = 6 \cdot 2 \cdot x^{\frac{2}{3}} \cdot x^{\frac{3}{4}}$

$= 6 \cdot 2 \cdot x^{\frac{2}{3}} \cdot x^{\frac{3}{4}}$ Use $x^m \cdot x^n = x^{m+n}$.

$= 12x^{\frac{17}{12}}$ Simplify.

## Exercises

**Simplify each expression. Assume that all variables are positive.**

**1.** $5^{\frac{1}{3}} \cdot 5^{\frac{2}{3}}$

**2.** $(2y^{\frac{1}{4}})(3y^{\frac{1}{3}})$

**3.** $(-11)^{\frac{1}{3}} \cdot (-11)^{\frac{1}{3}} \cdot (-11)^{\frac{1}{3}}$

**4.** $-y^{\frac{2}{3}} y^{\frac{1}{5}}$

**5.** $5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}}$

**6.** $(-3x^{\frac{1}{6}})(7x^{\frac{2}{6}})$

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**Algebra 2: Guidelines for Applying Properties of Exponents**

| | **Products** | **Quotients** |
|---|---|---|
| **Bases are the same ›** | Add the exponents: $b^m \cdot b^n = b^{m+n}$ | Subtract the exponents: $\frac{b^m}{b^n} = b^{m-n} \quad (b \neq 0)$ (order matters!) |
| **Exponents are the same ›** | Multiply the bases: $a^n \cdot b^n = (ab)^n$ | Divide the bases: $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad (b \neq 0)$ |
| | | |
| **Power of a Power ›** | Multiply the exponents: $(a^m)^n = a^{mn}$ | |
| **Zero Power ›** | Except for zero itself, any real number raised to the zero power is one: $b^0 = 1 \qquad (b \neq 0)$ ($0^0$ is undefined) | |
| **Negative Exponent ›** | The negative exponent means one over the positive power: $b^{-n} = \frac{1}{b^n}$ and $\frac{1}{b^{-n}} = b^n \qquad (b \neq 0)$ Example with a factor that is *not* exponentiated: $ab^{-n} = \frac{a}{b^n}$ and $\frac{a}{b^{-n}} = ab^n$ | |
| **Rewriting Radicals ›** | The root is the denominator and the power is the numerator: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ | |

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# 6-4 Practice

*Form G*

**Simplify each expression.**

**1.** $125^{\frac{1}{3}}$

**2.** $64^{\frac{1}{2}}$

**3.** $32^{\frac{1}{5}}$

**4.** $7^{\frac{1}{2}} \cdot 7^{\frac{1}{2}}$

**5.** $(-5)^{\frac{1}{3}} \cdot (-5)^{\frac{1}{3}} \cdot (-5)^{\frac{1}{3}}$

**6.** $3^{\frac{1}{2}} \cdot 75^{\frac{1}{2}}$

**7.** $11^{\frac{1}{3}} \cdot 11^{\frac{1}{3}} \cdot 11^{\frac{1}{3}}$

**8.** $7^{\frac{1}{2}} \cdot 28^{\frac{1}{2}}$

**9.** $8^{\frac{1}{4}} \cdot 32^{\frac{1}{4}}$

**10.** $12^{\frac{1}{2}} \cdot 27^{\frac{1}{2}}$

**11.** $12^{\frac{1}{3}} \cdot 45^{\frac{1}{3}} \cdot 50^{\frac{1}{3}}$

**12.** $18^{\frac{1}{2}} \cdot 98^{\frac{1}{2}}$

**Write each expression in radical form.**

**13.** $x^{\frac{4}{3}}$

**14.** $(2y)^{\frac{1}{3}}$

**15.** $a^{1.5}$

**16.** $b^{\frac{1}{5}}$

**17.** $z^{\frac{2}{3}}$

**18.** $(ab)^{\frac{1}{4}}$

**19.** $m^{2.4}$

**20.** $t^{-\frac{2}{7}}$

**21.** $a^{-1.6}$

**Write each expression in exponential form.**

**22.** $\sqrt{x^3}$

**23.** $\sqrt[3]{m}$

**24.** $\sqrt{5y}$

**25.** $\sqrt[3]{2y^2}$

**26.** $(\sqrt[4]{b})^3$

**27.** $\sqrt{-6}$

**28.** $\sqrt{(6a)^4}$

**29.** $\sqrt[5]{n^4}$

**30.** $\sqrt[4]{(5ab)^3}$

**31.** The rate of inflation $i$ that raises the cost of an item from the present value $P$ to the future value $F$ over $t$ years is found using the formula $i = \left(\frac{F}{P}\right)^{\frac{1}{t}} - 1$.

Round your answers to the nearest tenth of a percent.

**a.** What is the rate of inflation for which a television set costing $1000 today will become one costing $1500 in 3 years?

**b.** What is the rate of inflation that will result in the price $P$ doubling (that is, $F = 2P$) in 10 years?

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# 6-4 Practice (continued)

*Form G*

**Write each expression in simplest form. Assume that all variables are positive.**

**32.** $(81^{\frac{1}{4}})^4$

**33.** $(32^{\frac{1}{5}})^5$

**34.** $(256^4)^{\frac{1}{4}}$

**35.** $7^0$

**36.** $8^{\frac{2}{3}}$

**37.** $(-27)^{\frac{2}{3}}$

**38.** $x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}$

**39.** $2y^{\frac{1}{2}} \cdot y$

**40.** $(8^2)^{\frac{1}{3}}$

**41.** $3.6^0$

**42.** $\left(\frac{1}{16}\right)^{\frac{1}{4}}$

**43.** $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

**44.** $\sqrt[8]{0}$

**45.** $(3x^{\frac{1}{2}})(4x^{\frac{2}{3}})$

**46.** $\dfrac{12y^{\frac{1}{3}}}{4y^{\frac{1}{2}}}$

**47.** $(3a^{\frac{1}{2}}b^{\frac{1}{3}})^2$

**48.** $(y^{\frac{2}{3}})^{-9}$

**49.** $(a^{\frac{2}{3}}b-\frac{1}{2})^{-6}$

**50.** $y^{\frac{2}{5}} \cdot y^{\frac{3}{8}}$

**51.** $\left(\dfrac{x^{\frac{4}{7}}}{x^{\frac{2}{3}}}\right)$

**52.** $(2a^{\frac{1}{4}})^3$

**53.** $81^{-\frac{1}{2}}$

**54.** $(2x^{\frac{2}{5}})(6x^{\frac{1}{4}})$

**55.** $(9x^4y^{-2})^{\frac{1}{2}}$ 12

**56.** $\left(\dfrac{27x^6}{64y^4}\right)^{\frac{1}{3}}$

**57.** $\dfrac{x^{\frac{1}{2}}y^{\frac{2}{3}}}{x^{\frac{1}{3}}y^{\frac{1}{2}}}$

**58.** $y^{\frac{5}{8}} \div y^{\frac{1}{2}}$

**59.** $x^{\frac{1}{4}} \cdot x^{\frac{1}{6}} \cdot x^{\frac{1}{3}}$

**60.** $\left(\dfrac{x^{-\frac{1}{3}}y}{x^{\frac{2}{3}}y^{-\frac{1}{2}}}\right)^2$

**61.** $\left(\dfrac{12x^8}{75y^{10}}\right)^{\frac{1}{2}}$

**62.** In a test kitchen, researchers have measured the radius of a ball of dough made with a new quick-acting yeast. Based on their data, the radius $r$ of the dough ball, in centimeters, is given by $r = 5(1.05)^{\frac{t}{3}}$ after $t$ minutes. Round the answers to the following questions to the nearest tenth of a cm.

**a.** What is the radius after 5 minutes?

**b.** What is the radius after 20 minutes?

**c.** What is the radius after 43 minutes?