Algebra 2 – Warm Up – Apr 2, 2014 Name: _____

a. What are the expressions $w^{-\frac{5}{8}}$ and $w^{0.2}$ in radical form?

b. What are the expressions $\sqrt[4]{x^3}$ and $(\sqrt[5]{y})^4$ in exponential form?

c. Reasoning Refer to the definition of rational exponent. Explain the need for the restriction that $a \neq 0$ if m is negative.

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6-4 Reteaching

You can simplify a number with a rational exponent by converting the expression to a radical expression:

$x^{\frac{1}{n}} = \sqrt[n]{x}$, for $n > 0$ $\qquad 9^{\frac{1}{2}} = \sqrt[2]{9} = 3$ $\qquad 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

You can simplify the product of numbers with rational exponents *m* and *n* by raising the number to the sum of the exponents using the rule

$$a^m \cdot a^n = a^{m+n}$$

Problem

What is the simplified form of each expression?

a. $36^{\frac{1}{4}} \cdot 36^{\frac{1}{4}}$

$36^{\frac{1}{4}} \cdot 36^{\frac{1}{4}} = 36^{\frac{1}{4}+\frac{1}{4}}$ Use $a^m \cdot a^n = a^{m+n}$.

$= 36^{\frac{1}{2}}$ Add.

$= \sqrt[2]{36}$ Use $x^{\frac{1}{n}} = \sqrt[n]{x}$.

$= 6$ Simplify.

b. Write $(6x^{\frac{2}{3}})(2x^{\frac{3}{4}})$ in simplified form.

$(6x^{\frac{2}{3}})(2x^{\frac{3}{4}}) = 6 \cdot 2 \cdot x^{\frac{2}{3}} \cdot x^{\frac{3}{4}}$ Commutative and Associative Properties of Multiplication

$= 6 \cdot 2 \cdot x^{\frac{2}{3}} \cdot x^{\frac{3}{4}}$ Use $x^m \cdot x^n = x^{m+n}$.

$= 12x^{\frac{17}{12}}$ Simplify.

Exercises

Simplify each expression. Assume that all variables are positive.

1. $5^{\frac{1}{3}} \cdot 5^{\frac{2}{3}}$ **2.** $(2y^{\frac{1}{4}})(3y^{\frac{1}{3}})$ **3.** $(-11)^{\frac{1}{3}} \cdot (-11)^{\frac{1}{3}} \cdot (-11)^{\frac{1}{3}}$

4. $-y^{\frac{2}{3}}y^{\frac{1}{5}}$ **5.** $5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}}$ **6.** $(-3x^{\frac{1}{6}})(7x^{\frac{2}{6}})$

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6-4 Reteaching (continued)

To write an expression with rational exponents in simplest form, simplify all exponents and write every exponent as a positive number using the following rules for $a \neq 0$ and rational numbers m and n:

$$a^{-n} = \frac{1}{a^n} \qquad \frac{1}{a^{-m}} = a^m \qquad (a^m)^n = a^{mn} \qquad (ab)^m = a^m b^m$$

Problem

What is $(8x^9y^{-3})^{-\frac{2}{3}}$ in simplest form?

$(8x^9y^{-3})^{-\frac{2}{3}} = (2^3x^9y^{-3})^{-\frac{2}{3}}$ Factor any numerical coefficients.

$= (2^3)^{-\frac{2}{3}}(x^9)^{-\frac{2}{3}}(y^{-3})^{-\frac{2}{3}}$ Use the property $(ab)^m = a^m b^m$

$= 2^{-2}x^{-6}y^2$ Multiply exponents, using the property $(a^m)^n = a^{mn}$.

$= \frac{y^2}{2^2x^6}$ Write every exponent as a positive number.

$= \frac{y^2}{4x^6}$ Simplify.

Exercises

Write each expression in simplest form. Assume that all variables are positive.

7. $\left(16x^2y^8\right)^{-\frac{1}{2}}$ **8.** $\left(z^{-3}\right)^{\frac{1}{9}}$ **9.** $\left(2x^{\frac{1}{4}}\right)^4$

10. $\left(25x^{-6}y^2\right)^{\frac{1}{2}}$ **11.** $\left(8a^{-3}b^9\right)^{\frac{2}{3}}$ **12.** $\left(\frac{16z^4}{25x^8}\right)^{\frac{-1}{2}}$

13. $\left(\frac{x^2}{y^{-1}}\right)^{\frac{1}{5}}$ **14.** $\left(27m^9n^{-3}\right)^{-\frac{2}{3}}$ **15.** $\left(\frac{32r^2}{2s^4}\right)^{\frac{1}{4}}$

16. $\left(9z^{10}\right)^{\frac{3}{2}}$ **17.** $(-243)^{-\frac{1}{5}}$ **18.** $\left(\frac{x^{\frac{2}{5}}}{y^{\frac{1}{2}}}\right)^{10}$

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6-4 Practice

Form K

Rational Exponents

Simplify each expression.

1. $16^{\frac{1}{4}}$

$\sqrt[4]{16}$

2. $(-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}}$

3. $5^{\frac{1}{2}} \cdot 45^{\frac{1}{2}}$

Write each expression in radical form.

4. $x^{\frac{1}{4}}$

5. $x^{\frac{4}{5}}$

6. $x^{\frac{2}{9}}$

Write each expression in exponential form.

7. $\sqrt[3]{2}$

8. $\sqrt[3]{2x^2}$

9. $\sqrt[3]{(2x)^2}$

10. Bone loss for astronauts may be prevented with an apparatus that rotates to simulate gravity. In the formula $N = \frac{a^{0.5}}{2\pi r^{0.5,}}$, N is the rate of rotation in revolutions per second, a is the simulated acceleration in m/s^2, and r is the radius of the apparatus in meters. How fast would an apparatus with the following radii have to rotate to simulate the acceleration of 9.8 m/s^2 that is due to Earth's gravity?

a. r = 1.7 m

b. r = 3.6 m

c. r = 5.2 m

d. Reasoning Would an apparatus with radius 0.8 m need to spin faster or slower than the one in part (a)?

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6-4 Practice (continued) *Form K*

Simplify each number.

11. $(-216)^{\frac{1}{3}}$

$\sqrt[3]{-216}$

12. $243^{1.2}$

13. $32^{-0.4}$

Find each product or quotient. To start, rewrite the expression using exponents.

14. $\left(\sqrt[4]{6}\right)\left(\sqrt[3]{6}\right)$

$= \left(6^{\frac{1}{4}}\right)\left(6^{\frac{1}{3}}\right)$

15. $\dfrac{\sqrt[5]{x^2}}{\sqrt[10]{x^2}}$

16. $\sqrt{20} \cdot \sqrt[3]{135}$

Simplify each number.

17. $(125)^{\frac{2}{3}}$

18. $(216)^{\frac{2}{3}}(216)^{\frac{2}{3}}$

19. $(-243)^{\frac{2}{5}}$

Write each expression in simplest form. Assume that all variables are positive.

20. $\left(16x^{-8}\right)^{\frac{3}{4}}$

21. $\left(8x^{15}\right)^{-\frac{1}{3}}$

22. $\left(\dfrac{x^2}{x^{-10}}\right)^{\frac{1}{3}}$

23. Error Analysis Explain why the following simplification is incorrect. What is the correct simplification?

$$5\left(4-5^{\frac{1}{2}}\right)$$

$$= 5(4) - 5\left(5^{\frac{1}{2}}\right) = 20 - 25^{\frac{1}{2}} = 15$$

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6-5 Reteaching

Equations containing radicals can be solved by isolating the radical on one side of the equation, and then raising both sides to the same power that would undo the radical.

Problem

What is the solution of the radical equation? $2\sqrt{2x+2}-2=10$

$2\sqrt{2x+2}-2=10$	
$2\sqrt{2x+2}=12$	Add 2 to each side.
$\sqrt{2x+2}=6$	Divide each side by 2.
$(\sqrt{2x+2})^2=6^2$	Square each side to undo the radical.
$2x+2=36$	Simplify.
$2x=34$	Subtract 2 from each side.
$x=17$	Divide each side by 2.

Check the solution in the original equation.

Check

$2\sqrt{2x+2}-2=10$	Write the original equation.
$2\sqrt{2(17)+2}-2 \stackrel{?}{=} 10$	Replace x by 17.
$2\sqrt{36}-2 \stackrel{?}{=} 10$	Simplify.
$12-2 \stackrel{?}{=} 10$	
$10=10$	

The solution is 17.

Exercises

Solve. Check your solutions.

1. $x^{\frac{1}{2}}=13$

2. $3\sqrt{2x}=12$

3. $\sqrt{3x}+5=11$

4. $(3x+4)^{\frac{1}{2}}-1=4$

5. $(6-x)^{\frac{1}{2}}+2=5$

6. $\sqrt{3x+13=4}$

7. $(x+2)^{\frac{1}{2}}-5=0$

8. $\sqrt{3-2x}-2=3$

9. $\sqrt[3]{5x+2}-3=0$

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6-5 Reteaching (continued)

An extraneous solution may satisfy equations in your work, but it does not make the original equation true. Always check possible solutions in the original equation.

Problem

What is the solution? Check your results. $\sqrt{17-x}-3=x$

$\sqrt{17-x}-3=x$

$\sqrt{17-x}=x+3$ Add 3 to each side to get the radical alone on one side of the equal sign.

$(\sqrt{17-x})^2=(x+3)^2$ Square each side.

$17-x=x^2+6x+9$

$0=x^2+7x-8$ Rewrite in standard form.

$0=(x-1)(x+8)$ Factor.

$x-1=0$ or $x+8=0$ Set each factor equal to 0 using the Zero Product Property.

$x=1$ or $x=-8$

Check

$\sqrt{17-x}-3 \stackrel{?}{=} x$

$\sqrt{17-1}-3 \stackrel{?}{=} 1$

$\sqrt{16}-3 \stackrel{?}{=} 1$

$1=1$ ✓

$\sqrt{17-x}-3 \stackrel{?}{=} x$

$\sqrt{17-(-8)}-3 \stackrel{?}{=} -8$

$\sqrt{25}-3 \stackrel{?}{=} 8$

$2 \neq -8$

The only solution is 1.

Exercises

Solve. Check for extraneous solutions.

10. $\sqrt{5x+1}=\sqrt{4x+3}$

11. $\sqrt{x^2+3}=x+1$

12. $\sqrt{3x}=\sqrt{x+6}$

13. $x=\sqrt{x+7}+5$

14. $x-3\sqrt{x}-4=0$

15. $\sqrt{x+2}=x-4$

16. $\sqrt{2x-10}=x-5$

17. $\sqrt{3x-6}=2-x$

18. $\sqrt{x-1}+7=x$

19. $\sqrt{5x+1}=\sqrt{3x+15}$

20. $\sqrt{x+9}=x+7$

21. $x-\sqrt{x+2}=40$

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6-5 Practice

Form K

Solve. To start, rewrite the equation to isolate the radical.

1. $\sqrt{x+2}-2=0$

$\sqrt{x+2}=2$

2. $\sqrt{2x+3}-7=0$

3. $2+\sqrt{3x-2}=6$

Solve.

4. $2(x-2)^{\frac{2}{3}}=50$

5. $2(x+3)^{\frac{3}{2}}=54$

6. $(6x-5)^{\frac{1}{3}}+3=-2$

7. A The formula $d=2\sqrt{\frac{V}{\pi h}}$ relates the diameter *d*, in units, of cylinder to its volume *V*, in cubic units, and its height *h*, in units. A cylindrical can has a diameter of 3 in. and a height of 4 in. What is the volume of the can to the nearest cubic inch?

8. Writing Explain the difference between a radical equation and a polynomial equation.

9. Reasoning If you are solving $4(x+3)^{\frac{3}{4}}=7$, do you need to use the absolute value to solve for *x*? Why or why not?

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6-5 Practice (continued) *Form K*

Solve. Check for extraneous solutions. First, isolate the radical, then square each side of the equation.

10. $\sqrt{4x+5} = x+2$

$\left(\sqrt{4x+5}\right)^2 = (x+2)^2$

11. $\sqrt{-3x-5} - 3 = x$

12. $\sqrt{x+7} + 5 = x$

13. $\sqrt{2x-7} = \sqrt{x+2}$

$\left(\sqrt{2x-7}\right)^2 = \left(\sqrt{x+2}\right)^2$

14. $\sqrt{3x+2} - \sqrt{2x+7} = 0$

15. $\sqrt{2x+4} - 2 = \sqrt{x}$

16. Find the solutions of $\sqrt{x+2} = x$.

a. Are there any extraneous solutions?

b. Reasoning How do you know the answer to part (a)?

17. A floor is made up of hexagon-shaped tiles. Each hexagon tile has an area of 1497 cm^2. What is the length of each side of the hexagon? (*Hint:* Six equilateral triangles make one hexagon.)