

7-4 Reteaching

Properties of Logarithms

You can write a logarithmic expression containing more than one logarithm as a single logarithm as long as the bases are equal. You can write a logarithm that contains a number raised to a power as a logarithm with the power as a coefficient. To understand the following properties, remember that logarithms are powers.

Name	Formula	Why?
Product Property	$\log_b mn = \log_b m + \log_b n$	When you multiply two powers, you add the exponents. Example: $2^6 \cdot 2^2 = 2^{(6+2)} = 2^8$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$	When you divide two powers, you subtract the exponents. Example: $\frac{2^6}{2^2} = 2^{(6-2)} = 2^4$
Power Property	$\log_b m^n = n \log_b m$	When you raise a power to a power, you multiply the exponents. Example: $(2^6)^2 = 2^{(6 \cdot 2)} = 2^{12}$

Problem

What is $2 \log_2 6 - \log_2 9 + \frac{1}{3} \log_2 27$ written as a single logarithm?

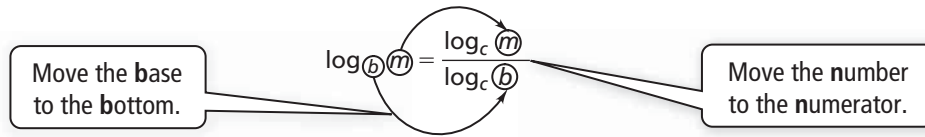
$$\begin{aligned}
 2 \log_2 6 - \log_2 9 + \frac{1}{3} \log_2 27 &= \log_2 6^2 - \log_2 9 + \log_2 27^{\frac{1}{3}} && \text{Use the Power Property twice.} \\
 &= \log_2 36 - \log_2 9 + \log_2 3 && 6^2 = 36, 27^{\frac{1}{3}} = \sqrt[3]{27} = 3 \\
 &= (\log_2 36 - \log_2 9) + \log_2 3 && \text{Group two of the logarithms. Use} \\
 & && \text{order of operations.} \\
 &= \log_2 \frac{36}{9} + \log_2 3 && \text{Quotient Property} \\
 &= \log_2 \left(\frac{36}{9} \cdot 3 \right) && \text{Product Property} \\
 &= \log_2 12 && \text{Simplify.}
 \end{aligned}$$

As a single logarithm, $2 \log_2 6 - \log_2 9 + \frac{1}{3} \log_2 27 = \log_2 12$.

7-4 **Reteaching** (continued)

Properties of Logarithms

To evaluate logarithms with any base, you can rewrite the logarithm as a quotient of two logarithms with the same base.



Problem

What is $\log_4 8$ written as a quotient of two logarithms with base 2? Simplify your answer, if possible.

$$\log_4 8 = \frac{\log_2 8}{\log_2 4}$$

The base is 4 and the number is 8. Move the base to the bottom and the number to the numerator.

$$= \frac{3}{2}$$

Evaluate the logarithms in the numerator and the denominator.

Exercises

Write each logarithmic expression as a single logarithm.

1. $\log_3 13 + \log_3 3$ **$\log_3 39$**
2. $2 \log x + \log 5$ **$\log 5x^2$**
3. $\log_4 2 - \log_4 6$ **$\log_4 \frac{1}{3}$**
4. $3 \log_3 3 - \log_3 3$ **$\log_3 9$, or 2**
5. $\log_5 8 + \log_5 x$ **$\log_5 8x$**
6. $\log 2 - 2 \log x$ **$\log \frac{2}{x^2}$**
7. $\log_2 x + \log_2 y$ **$\log_2 xy$**
8. $3 \log_7 x - 5 \log_7 y$ **$\log_7 \frac{x^3}{y^5}$**
9. $4 \log x + 3 \log x$ **$\log x^7$**
10. $\log_5 x + 3 \log_5 y$ **$\log_5 xy^3$**
11. $3 \log_2 x - \log_2 y$ **$\log_2 \frac{x^3}{y}$**
12. $\log_2 16 - \log_2 8$ **$\log_2 2$, or 1**

Write each logarithm as a quotient of two common logarithms. Simplify your answer, if possible. (*Hint: Common logarithms are logarithms with base 10.*)

13. $\log_4 12$ **$\frac{\log 12}{\log 4}$**
14. $\log_2 1000$ **$\frac{3}{\log 2}$**
15. $\log_5 16$ **$\frac{\log 16}{\log 5}$**
16. $\log_{11} 205$ **$\frac{\log 205}{\log 11}$**
17. $\log_9 32$ **$\frac{\log 32}{\log 9}$**
18. $\log_{100} 51$ **$\frac{\log 51}{2}$**

7-6 Additional Vocabulary Support

Natural Logarithms

Problem

What is the solution of the equation $3e^{4x} + 5 = 26$? Explain your work and check your solution.

$3e^{4x} + 5 = 26$	Write the original equation.
$3e^{4x} = 21$	Subtract 5 from each side.
$e^{4x} = 7$	Divide each side by 3.
$4x = \ln 7$	Rewrite in logarithmic form.
$x = 0.25 \ln 7$	Divide each side by 4.
$x \approx 0.486$	Use a calculator.

Check $3e^{4x} + 5 = 26$	Write the original equation.
$3e^{4(0.486)} + 5 = 26$	Substitute 0.486 for x .
$25.96 \approx 26$	Use a calculator.

Exercise

What is the solution of the equation $5e^{2x} - 2 = 12$? Explain your work and check your solution.

$5e^{2x} - 2 = 12$	Write the original equation.
$5e^{2x} = 14$	Add 2 to each side.
$e^{2x} = 2.8$	Divide each side by 5.
$2x = \ln 2.8$	Write in logarithmic form.
$x = 0.5 \ln 2.8$	Divide each side by 2.
$x \approx 0.515$	Use a calculator.

Check $5e^{2x} - 2 = 12$	Write the original equation.
$5e^{2(0.515)} - 2 = 12$	Substitute 0.515 for x.
$12.005 \approx 12$	Use a calculator.