2-4 Continuity and Discontinuity

Warm-Up

Where is the following function discontinuous? Why?

$$f(x) = \frac{x^2 - 9}{x - 3}$$

Continuous Function

- Informally – there are no “breaks” while sketching
  - Can be drawn without lifting the pencil from your paper

Name some continuous graphs

Types of Discontinuities

Removable (hole)

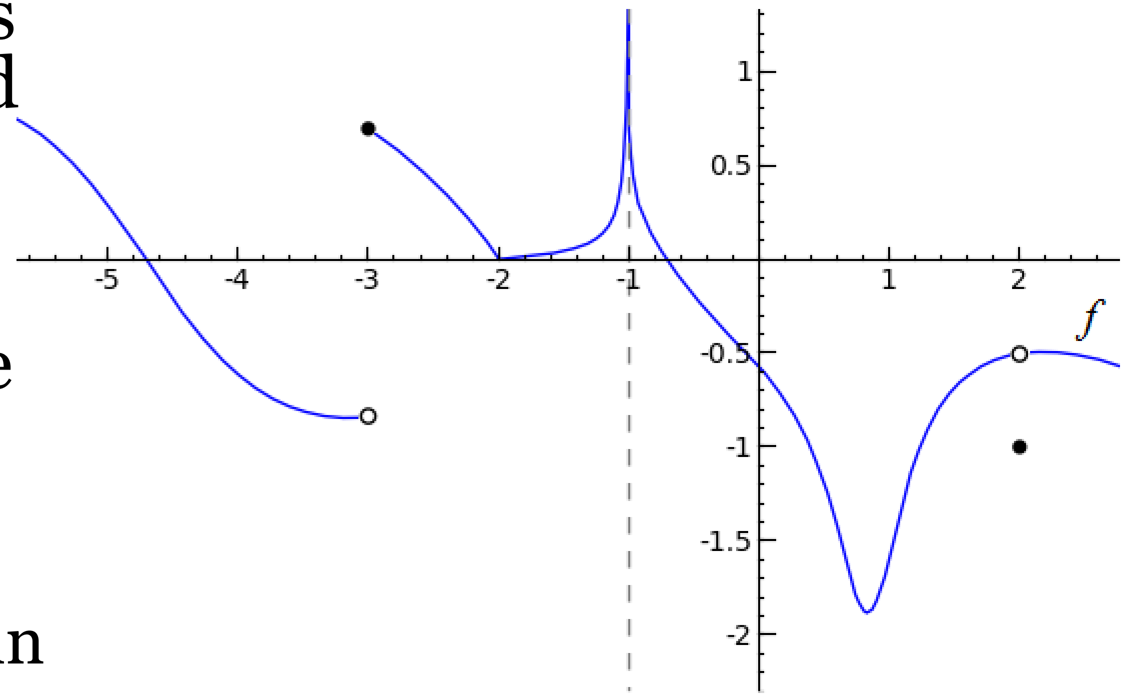
Come from the domain restrictions that can be factored out

Step (jump)

Usually a piecewise

Infinite (vertical asymptote)

Come from the domain restrictions that *cannot* be factored out



Let's Talk About…

Removable Discontinuities

From the Warm Up

$$f\left(x\right)=\frac{x^2-9}{x-3}$$

- No value for $f(3)$
- But there's a LIMIT as x approaches 3

Another Example

$$f\left(x\right)=\begin{cases}2x+1 & x\neq 0\\ 5 & x=0\end{cases}$$

- Has a value for $f(0)$
- But the LIMIT as x approaches 0 does not equal $f(0)$

Let's Talk About…

Step Discontinuities

Piecewise Functions

$$f(x) = \begin{cases} 2x+1 & x > 3 \\ x^2 & x \le 3 \end{cases}$$

- $\lim_{x\to 3^-} f(x)$ exists
- $\lim_{x\to 3^+} f(x)$ exists
- But is NO LIMIT as *x* approaches 3

Greatest Integer Function

$$f(x) = \quad x$$

- Definition
  - The greatest integer less than or equal to *x*
- Look at the Graph!
- At every integer value there is a jump which means NO LIMIT

Let's Talk About…

Infinite Discontinuities

Rational Functions

$$f\left(x\right)=\frac{x-2}{x-3}$$

Think about these one sided limits

- $\lim_{x\to 3^-} f(x)$
- $\lim_{x\to 3^+} f(x)$

Think about this problem

$$\lim_{x\to\infty} f(x)$$

Vertical Asymptotes –
Cause Discontinuities
Horizontal Asymptotes –
Describe what's happening as x gets infinitely large/small

Find the points of discontinuity of the function and identify the type

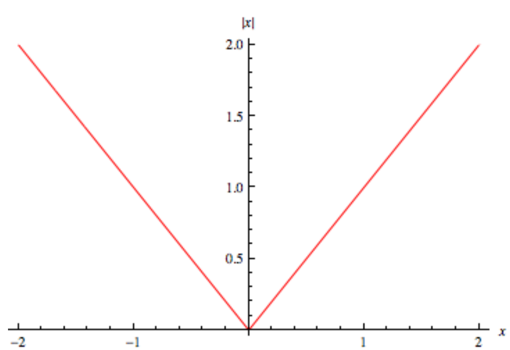
$$y = \frac{1}{(x-2)^2}$$

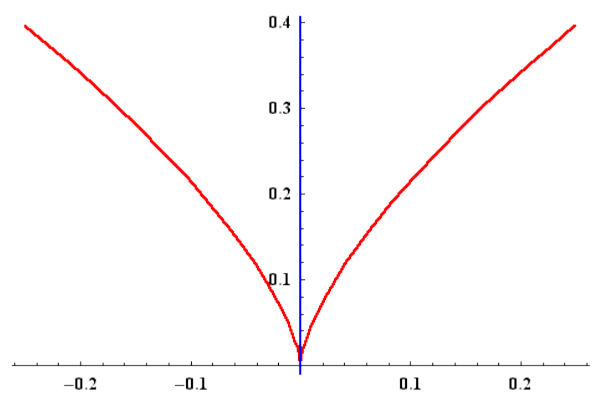
$$y = \frac{x-1}{x^2 - 4x + 3}$$

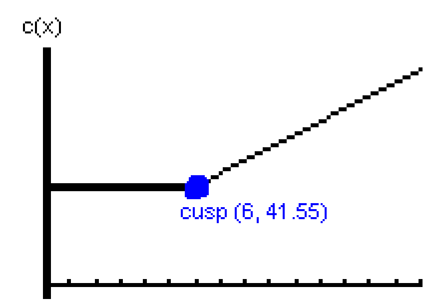
Cusp

A point on the graph at which the function is continuous but the derivative is discontinuous

A sharp point or an abrupt change in direction

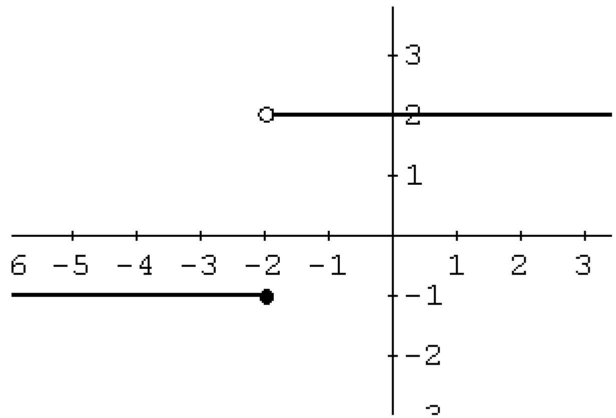






One-Sided Limits

Limits of the function coming just from the right (+) or the left (-) of the *c* value



$$\lim_{x \to -2^+} f(x) = 2 \qquad \lim_{x \to -2^-} f(x) = -1$$

Notation

Each part of a piecewise function is called a BRANCH

Graph on Calculator

$$f(x)=\begin{cases} x^2-1 & -1\le x<0 \\ 2x & 0<x<1 \\ 1 & x=1 \\ -2x+4 & 1<x<2 \\ 0 & 2<x<3 \end{cases}$$

- Does $f(-1)$ exist? Does $\lim_{x\to -1^+} f(x)$ exist?
- Does $\lim_{x\to -1^+} f(x) = f(-1)$?
- Repeat for x=1 and x=2
- For what values of x is the function continuous?
- Could you redefine the function to make it continuous for all $-1 \le x \le 3$? If not, at what points can you make it continuous?

Let’s look at some other examples

Exploration 2-4a:
Continuous and Discontinuous Functions

Find a value for *a* so that the given function is continuous

1. $f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2ax & x \geq 3 \end{cases}$

2. $f(x) = \begin{cases} 2x + 3 & x \leq 2 \\ ax + 1 & x > 2 \end{cases}$

3. $f(x) = \begin{cases} x^2 + x + a & x < 1 \\ x^3 & x \geq 1 \end{cases}$

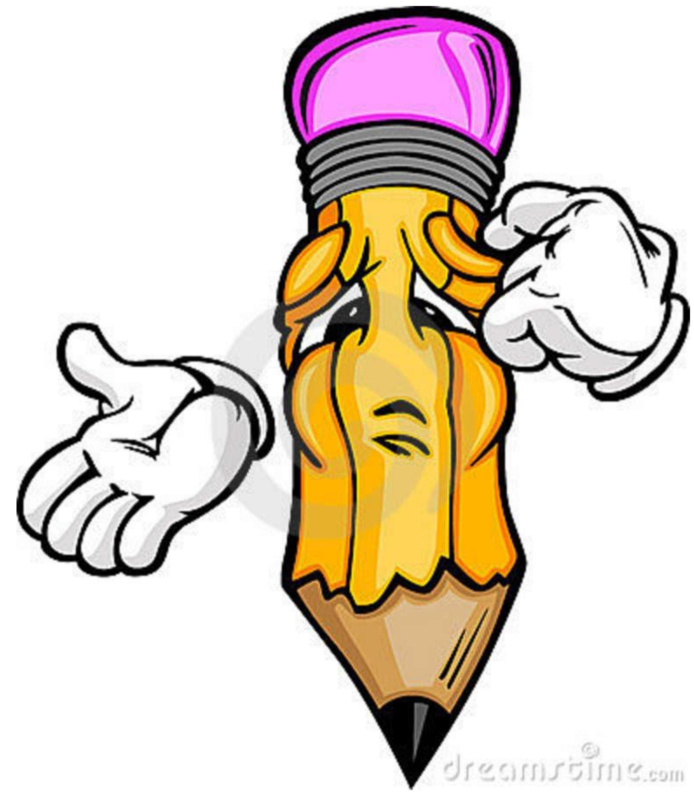
Why not try one more?! ☺

$$f(x) = \begin{cases} 5 - x^2 & x < 1 \\ 2ax - 1 & x \geq 1 \end{cases}$$

Let's Formally Sum up Continuity

Three things MUST be true in order for a function to be continuous at c

1. $f(c)$ exists
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$



If **ONE** is untrue,

the function has a discontinuity