

Big Ideas:

- Absolute values are never negative.
- All opposites have the same absolute value, so there can be two solutions to an absolute value equation.
- An absolute value equation can have **many solutions, two solutions, one solution** or **no solution**.
- Sometimes we can get an extraneous solution when solving absolute value equations.

Vocabulary & Notation:

Vocabulary	Definition
Absolute Value	Distance from zero on a real number line.
Extraneous Solution	A false solution that must be rejected because it does not make the equation true. Some techniques for solving equations will produce these false solutions, so we must always check our solutions to make sure they really work.

Notation	Spoken As
$ -5 $	"The absolute value of negative five"
$2 x $	"Two times the absolute value of x "

Rules:

- Absolute value symbols take priority in the order of operations. They are like parentheses, but you can't distribute a coefficient into an absolute value expression.

Examples:

a. $|8.9| = 8.9$

c. $|-7| = 7$

e. $\left|\frac{1}{2}\right| = \frac{1}{2}$

b. $|0| = 0$

d. $|-π| = π$

Section 1-6: Solving Absolute Value Equations

Skills. Solving an absolute value equation.

Examples:

- f. What numbers can make the following statement true?

$$|x| = 3$$

Since two numbers have an absolute value of 3, x could be either of those two numbers, so the solution is:

$$x = -3 \text{ or } x = 3$$

- g. What numbers can make the following statement true?

$$|x + 7| = 3$$

Since two numbers have an absolute value of 3, $x + 7$ could be either of those two numbers, so the solutions come from:

$$x + 7 = -3 \text{ or } x + 7 = 3$$

To find the solutions, solve *both* of the above equations to get:

$$x = -10 \text{ or } x = 4$$

- h. What numbers can make the following statement true?

$$|2x + 1| = -1$$

Since an absolute value can never be negative, this equation has no solution.

- i. What numbers can make the following statement true?

$$|4 - x| = 0$$

Since only zero has an absolute value of zero, the solution to this absolute value equation is the solution to

$$4 - x = 0$$

so $x = 4$.

Section 1-6: Solving Absolute Value Equations

j. Solve the absolute value equation and graph the solution.

$$3|x + 1| - 2 = 7$$

- Steps: ❶ isolate the absolute value expression
❷ write two equations to represent the two possible solutions
❸ solve, check and graph the two solutions

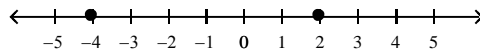
$$\begin{aligned} 3|x + 1| - 2 &= 7 \\ &+2 \quad +2 \\ 3|x + 1| &= 9 \\ \frac{3|x + 1|}{3} &= \frac{9}{3} \\ |x + 1| &= 3 \\ x + 1 &= -3 \quad \text{or} \quad x + 1 = 3 \\ x &= -4 \quad \text{or} \quad x = 2 \end{aligned}$$

Check:

$$\text{When } x = -4, 3|-4 + 1| - 2 = 3|-3| - 2 = 3 \cdot 3 - 2 = 9 - 2 = 7 \quad \checkmark$$

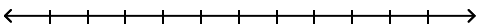
$$\text{When } x = 2, 3|2 + 1| - 2 = 3|3| - 2 = 3 \cdot 3 - 2 = 9 - 2 = 7 \quad \checkmark$$

Graph:

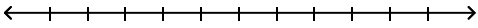


Practice. Solve and graph the following absolute value equations.

1. $|2 - x| + 1 = 5$ 

2. $5|x + 4| - 3 = 17$ 

3. $\frac{|x|}{2} = 12$ 

4. $x + |-7| = -8$ (Simplify this first! Is this an absolute value equation?) 

Section 1-6: Solving Absolute Value Equations

Example: Solving when there are variables inside and outside the absolute value expression.

$$\begin{aligned}
 \text{k. } 2|x - 1| + 3 &= 2x + 1 \\
 &\quad -3 \quad -3 \\
 2|x - 1| &= 2x - 2 \\
 \frac{2|x - 1|}{2} &= \frac{2x}{2} - \frac{2}{2} \\
 |x - 1| &= x - 1 \\
 x - 1 &= -(x - 1) \text{ or } x - 1 = x - 1 \\
 x - 1 &= -x + 1 \quad -x \quad -x \\
 +x \quad +x &\quad -1 = -1 \\
 2x - 1 &= 1 \quad \text{True for all reals} \\
 +1 \quad +1 & \\
 2x &= 2 \\
 x &= 1
 \end{aligned}$$

Check:

When $x = 1$: $2|1 - 1| + 3 = 2|0| + 3 = 2 \cdot 0 + 3 = 0 + 3 = 3 = 2(1) + 1 \checkmark$

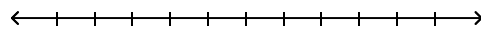
When $x = 0$: $2|0 - 1| + 3 = 2|-1| + 3 = 2 \cdot 1 + 3 = 2 + 3 = 5 \neq 2(0) + 1 \times$

Note: To test a *true for all reals* solution, choose ANY number that is not the other solution, and try it. If it does not work, then it is a false solution and we reject it.

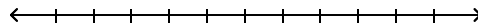
The only solution to this problem is $x=1$.

Practice. Solve, check and graph the following absolute value equations.

5. $|2x - 3| = 4x - 1$



6. $|2x + 5| = 3x + 4$



7. $|2x - 4| = 12$

